Magnetic effects of current

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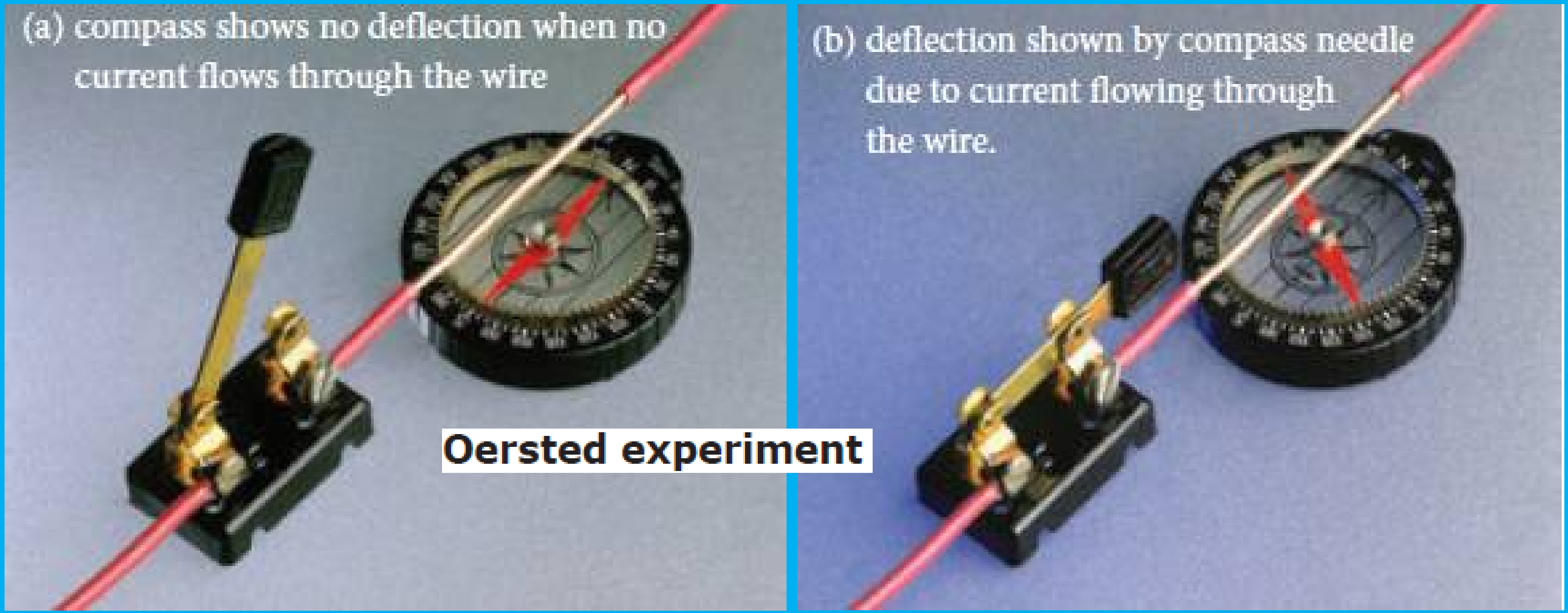
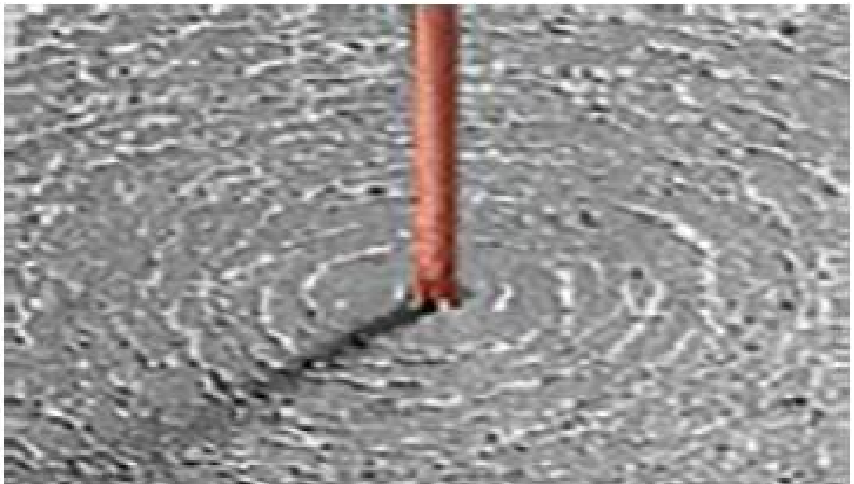
Oersted experiment



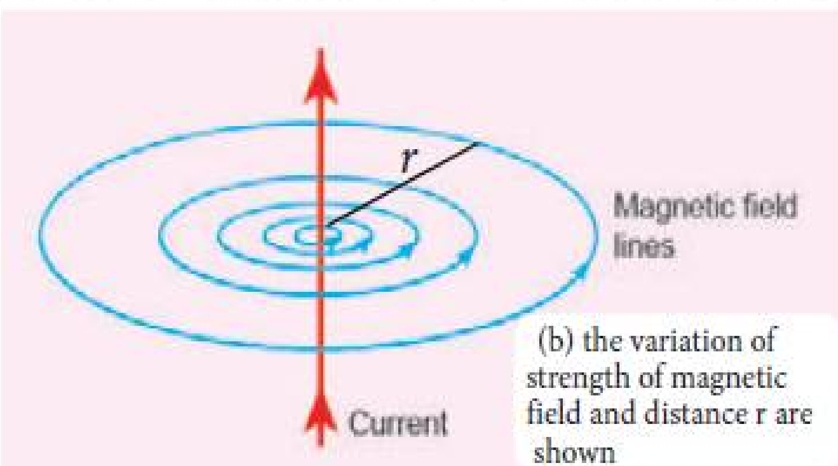
- The deflection of magnetic compass around current carrying conductor is due to presence of magnetic field. Magnetic needle deflects in opposite direction if the current is reversed.

Magnetic field around – straight conductor and circular coil

(a) Current carrying straight conductor:

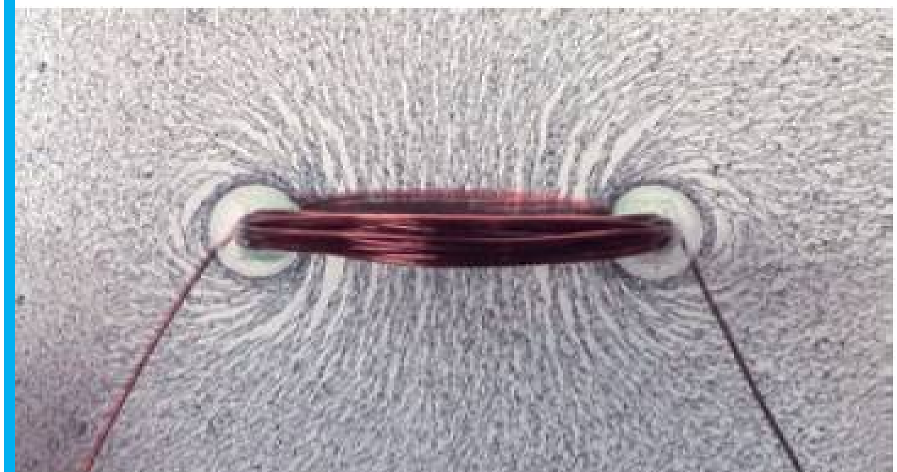


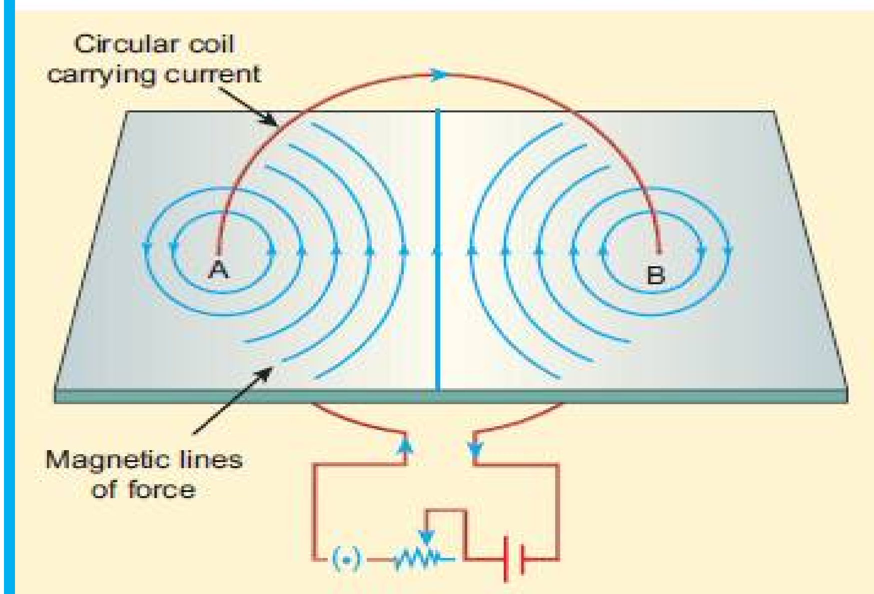
(a) the photograph of magnetic field lines curling around the conductor carrying current



(b) the variation of strength of magnetic field and distance r are shown

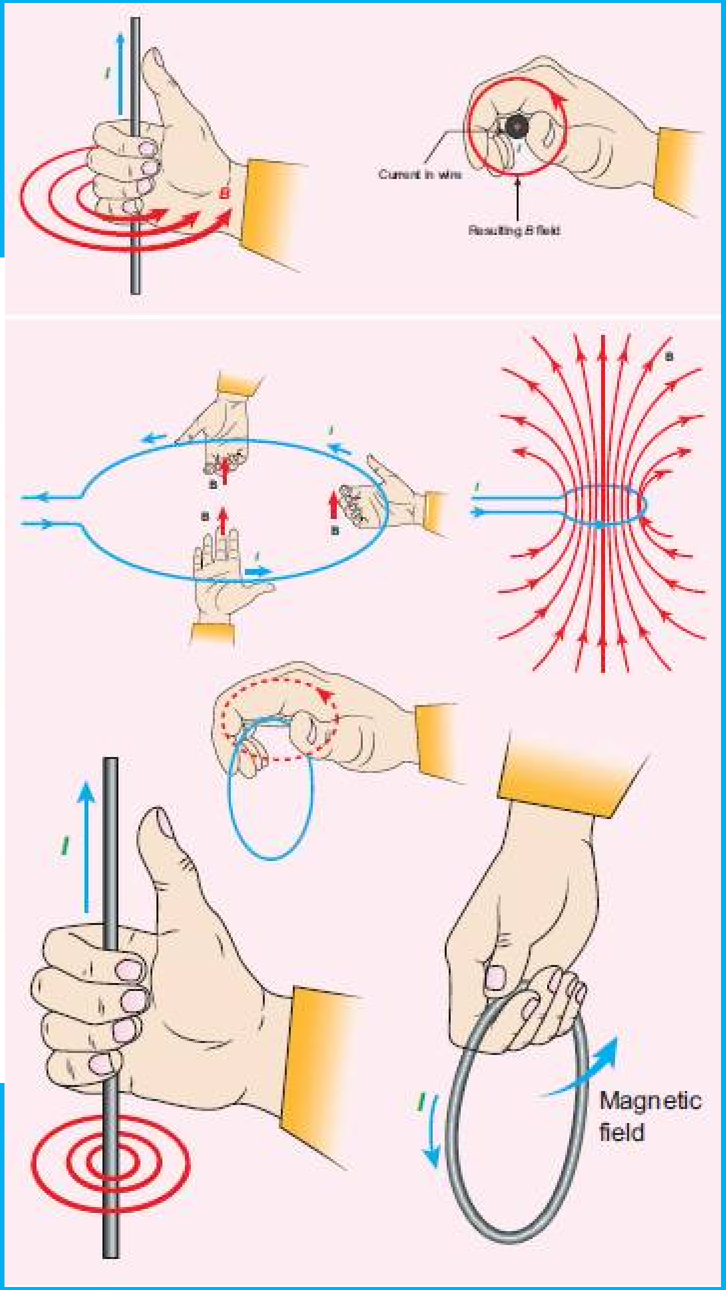
(b) Circular coil carrying current





Right hand thumb rule

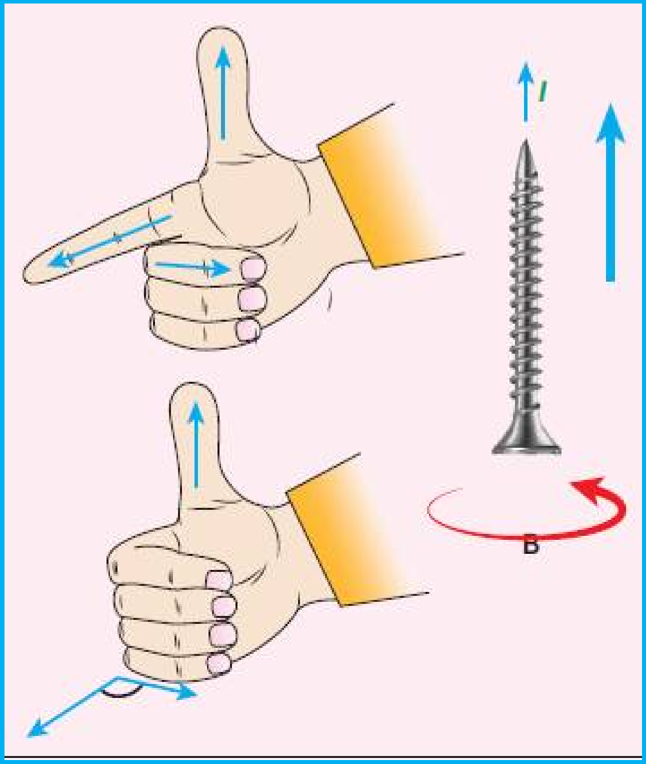
If we hold the current carrying conductor in our right hand such that the thumb points in the direction of current flow, then the fingers encircling the wire points in the direction of the magnetic field lines produced.



Maxwell's right hand cork screw rule

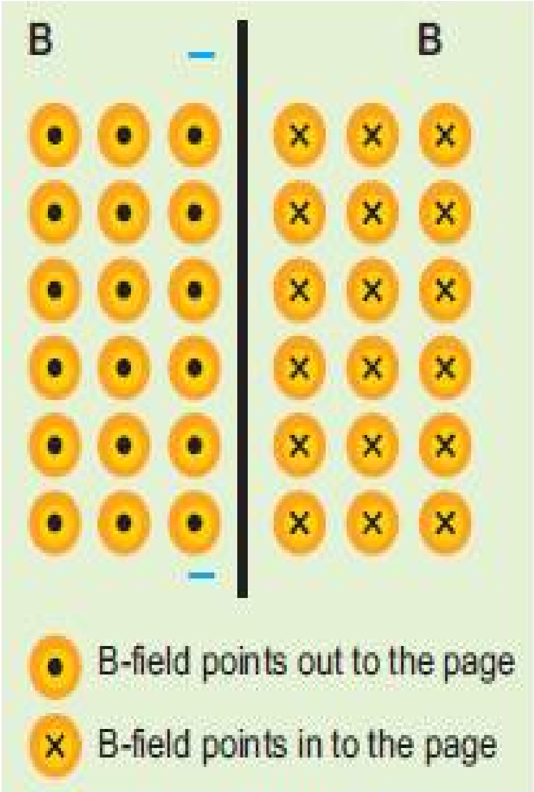
- This rule is used to determine the direction of the magnetic field.
- If we rotate a right-handed

screw using a screw driver, then the direction of current is same as the direction in which screw advances and the direction of rotation of the screw gives the direction of the magnetic field.

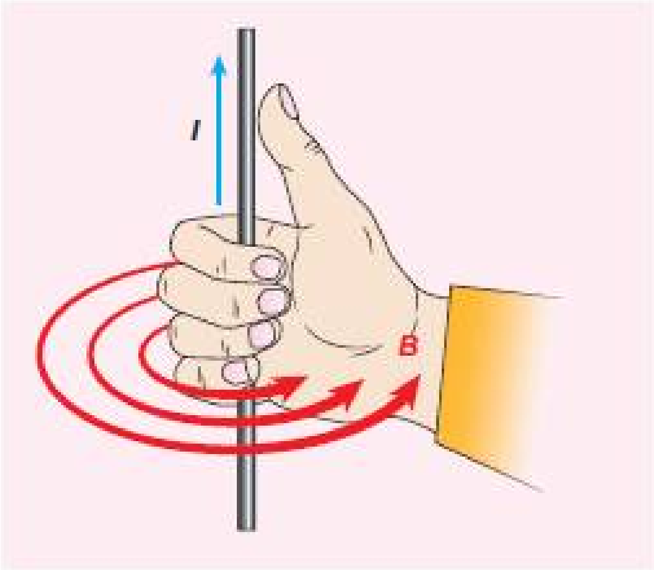


EXAMPLE 3.14

The magnetic field shown in the figure is due to the current carrying wire. In which direction does the current flow in the wire?.



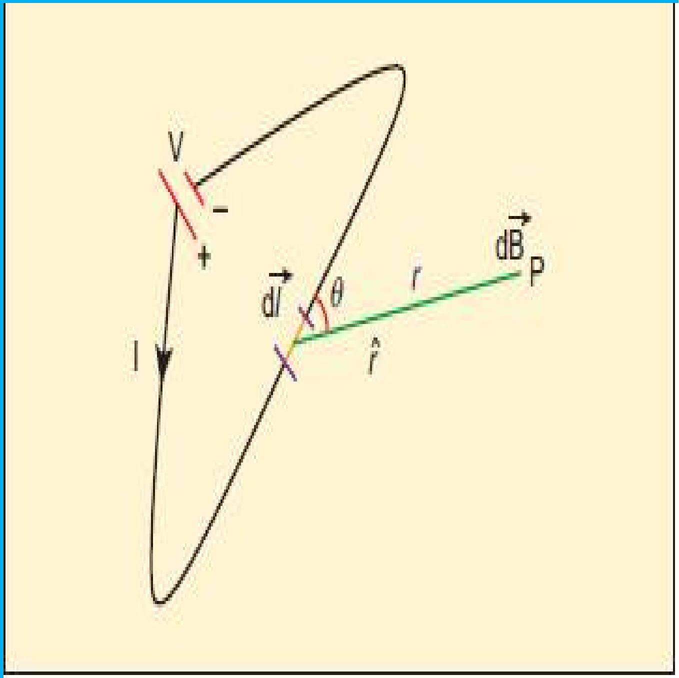
Solution



Using right hand rule, current flows upwards.

Biot-Savart's law

- Experimentally observed – the magnitude of magnetic field P at a distance r from the small elemental length taken on a conductor carrying current varies as



(i) directly as the strength of the current I
(ii) directly as the magnitude of the length element $d\vec{l}$
(iii) directly as the sine of the angle (say, θ) between $d\vec{l}$ and $\hat{r}$.
(iv) inversely as the square of the distance between the point P and length element $d\vec{l}$.

This is expressed as

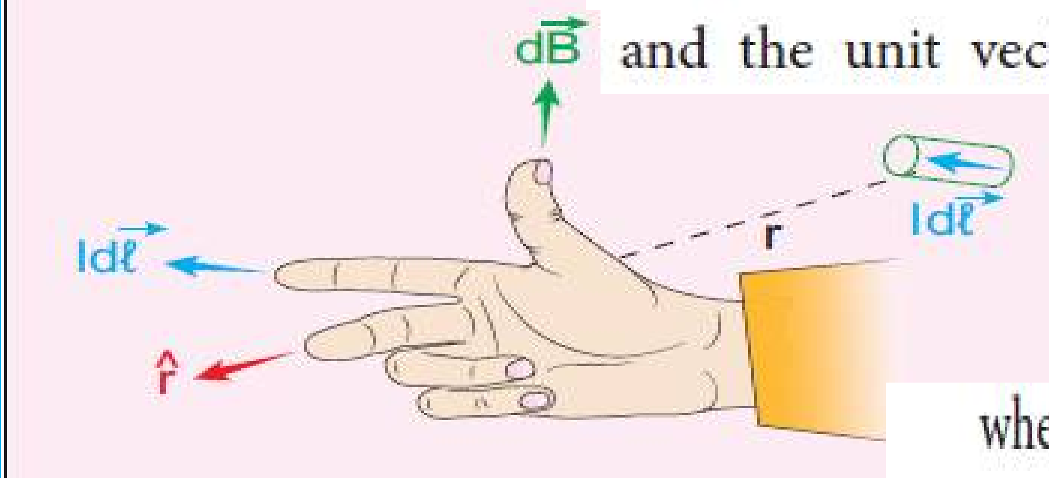
$$dB \propto \frac{Idl}{r^2}\sin\theta$$

$$dB = k\frac{I\,dl}{r^2}\sin\theta$$

where $k = \frac{\mu_\circ}{4\pi}$ in SI units and $k = 1$ in CGS units. In vector notation,

$$d\vec{B} = \frac{\mu_\circ}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2}$$

Here vector $d\vec{B}$ is perpendicular to both $I\,d\vec{l}$ (pointing the direction of current flow) and the unit vector $\hat{r}$ directed from $d\vec{l}$ toward point P



$$\vec{B} = \int d\vec{B} = \frac{\mu_\circ I}{4\pi}\int \frac{d\vec{l} \times \hat{r}}{r^2}$$

where the integral is taken over the entire current distribution.

Cases

1. If the point P lies on the conductor, then $\theta = 0^{\circ}$. Therefore, $d\vec{B}$ is zero.
2. If the point lies perpendicular to the conductor, then $\theta = 90^{\circ}$. Therefore, $d\vec{B}$ is maximum and is given by $d\vec{B} = \frac{I\,dl}{r^2}\hat{n}$ where $\hat{n}$ is the unit vector perpendicular to both $I\,d\vec{l}$ and $\hat{r}$

Similarities between Coulomb's law and Biot-Savort's law

Electric and magnetic fields

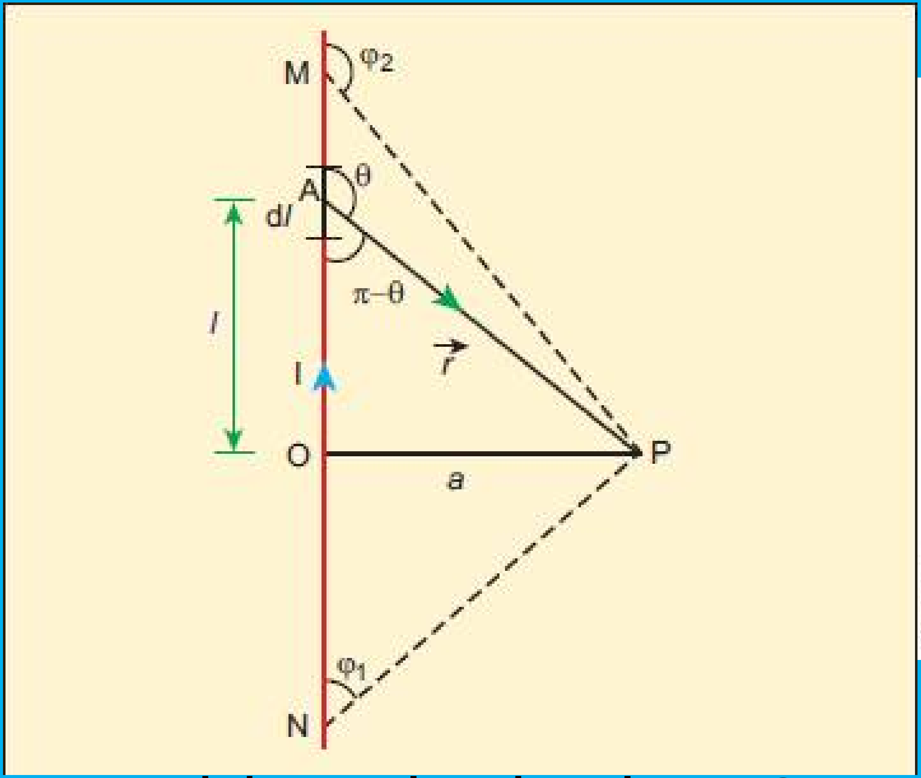
- obey inverse square law, so they are long range fields.
- obey the principle of superposition and are linear with respect to source. In magnitude,

$$E \propto q$$

$$B \propto Idl$$

Note that the exponent of charge q (source) and exponent of electric field E is unity. Similarly, the exponent of current element Idl (source) and exponent of magnetic field B is unity. In other words, electric field $\vec{E}$ is proportional only to charge (source) and not on higher powers of charge (q^2, q^3, etc). Similarly, magnetic field $\vec{B}$ is proportional to current element $Id\vec{l}$ (source) and not on square or cube or higher powers of current element. The cause and effect have linear relationship.

Long straight conductor – B field



$$d\vec{B} = \frac{\mu_\circ I}{4\pi}\frac{d\vec{l}}{r^2}\sin\theta\begin{pmatrix}\text{unit vector perpendicular}\\ \text{to } d\vec{l} \text{ and } \vec{r}\end{pmatrix}$$

The direction of the field is perpendicular to the plane of the paper and going into it. This can be determined by taking the cross product between two vectors $\vec{dl}$ and $\vec{r}$ (let it be $\hat{n}$).

In a right angle triangle, PAO

$$\tan(\pi-\theta) = \frac{a}{l}$$

$$l = -\frac{a}{\tan\theta} \quad (\text{since } \tan(\pi-\theta) = -\tan\theta)$$

$$l = -a\cot\theta \quad \textit{and} \quad r = a\,\mathrm{cosec}\,\theta$$

Differentiating

$$dl = a\,\mathrm{cosec}^2\theta\, d\theta$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\ (a\,\mathrm{cosec}^2\theta\ d\theta)}{(a\,\mathrm{cosec}\,\theta)^2}\sin\theta\,\hat{n} = \frac{\mu_0 I}{4\pi a}\sin\theta\,d\theta\,\hat{n}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\ (a\,\mathrm{cosec}^2\theta\ d\theta)}{(a\,\mathrm{cosec}\,\theta)^2}\sin\theta\,\hat{n}$$

This is the magnetic field at a point P due to the current in small elemental length. Note that we have expressed the magnetic field OP in terms of angular coordinate i.e. θ. Therefore, the net magnetic field at the point P which can be obtained by integrating $d\vec{B}$ by varying the angle from $\theta = \varphi_1$ to $\theta = \varphi_2$ is

$$\vec{B} = \frac{\mu_\circ I}{4\pi a}\int_{\varphi_1}^{\varphi_2}\sin\theta\,d\theta\,\hat{n} = \frac{\mu_\circ I}{4\pi a}(\cos\varphi_1 - \cos\varphi_2)\,\hat{n}$$

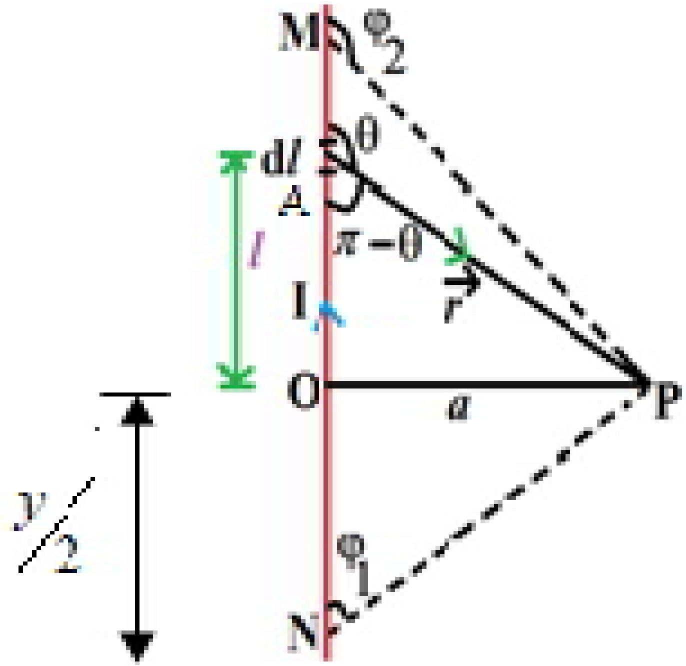
For a an infinitely long straight wire, $\varphi_1 = 0$ and $\varphi_2 = \pi$, the magnetic field is

$$\vec{B} = \frac{\mu_o I}{2\pi a}\hat{n}$$

Note that here $\hat{n}$ represents the unit vector from the point O to P.

EXAMPLE 3.15

Calculate the magnetic field at a point P which is perpendicular bisector to current carrying straight wire as shown in figure.



Solution

Let the length MN = y and the point P is on its perpendicular bisector. Let O be the point on the conductor as shown in figure.

Therefore, $OM = ON = \frac{y}{2}$, then

$$\cos\varphi_1 = \frac{\text{adjacent length}}{\text{hypotenuse length}} = \frac{ON}{PN} = \frac{\frac{y}{2}}{\sqrt{\frac{y^2}{4}+a^2}} = \frac{y}{\sqrt{y^2+4a^2}}$$

$$\cos(\pi-\varphi_2) = \frac{\text{adjacent length}}{\text{hypotenuse length}} = \frac{OM}{PM}$$

$$\Rightarrow -\cos\varphi_2 = \frac{\frac{y}{2}}{\sqrt{\frac{y^2}{4}+a^2}} \Rightarrow \cos\varphi_2 = -\frac{\frac{y}{2}}{\sqrt{\frac{y^2}{4}+a^2}}$$

Hence,

$$\vec{B} = \frac{\mu_\circ I}{4\pi a}\frac{2y}{\sqrt{y^2+4a^2}}\hat{n}$$

For long straight wire, $y \to \infty$,

$$\vec{B} = \frac{\mu_\circ I}{2\pi a}\hat{n}$$

EXAMPLE 3.16

Show that for a straight conductor, the magnetic field

$$\vec{B} = \frac{\mu_\circ I}{4\pi a}(\cos\varphi_1 - \cos\varphi_2)\hat{n}$$

$$= \frac{\mu_\circ I}{4\pi a}(\sin\theta_1 + \sin\theta_2)\hat{n}$$

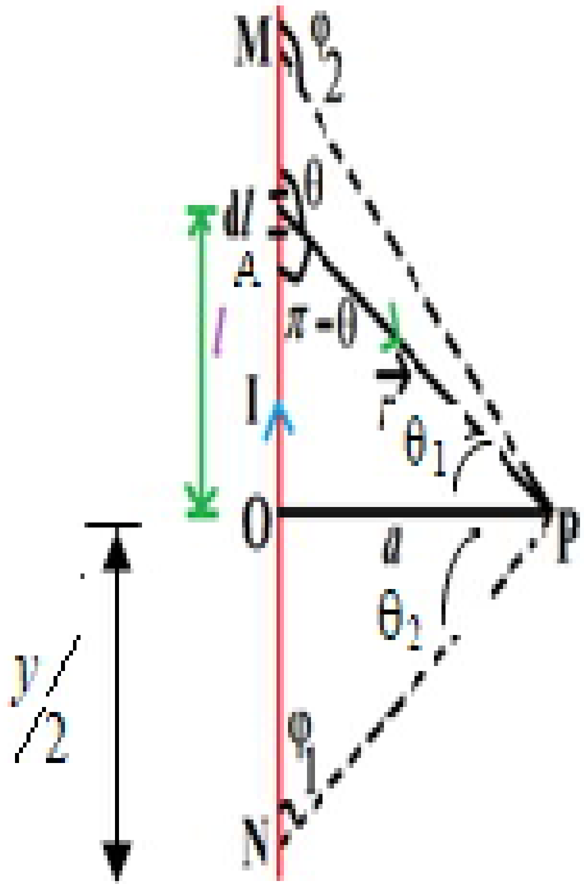
Solution:

In a right angle triangle OPN, let the angle $\angle OPN = \theta_1$ which implies, $\varphi_1 = \frac{\pi}{2} - \theta_1$ and also in a right angle triangle OPM, $\angle OPM = \theta_2$ which implies, $\varphi_2 = \frac{\pi}{2} + \theta_2$

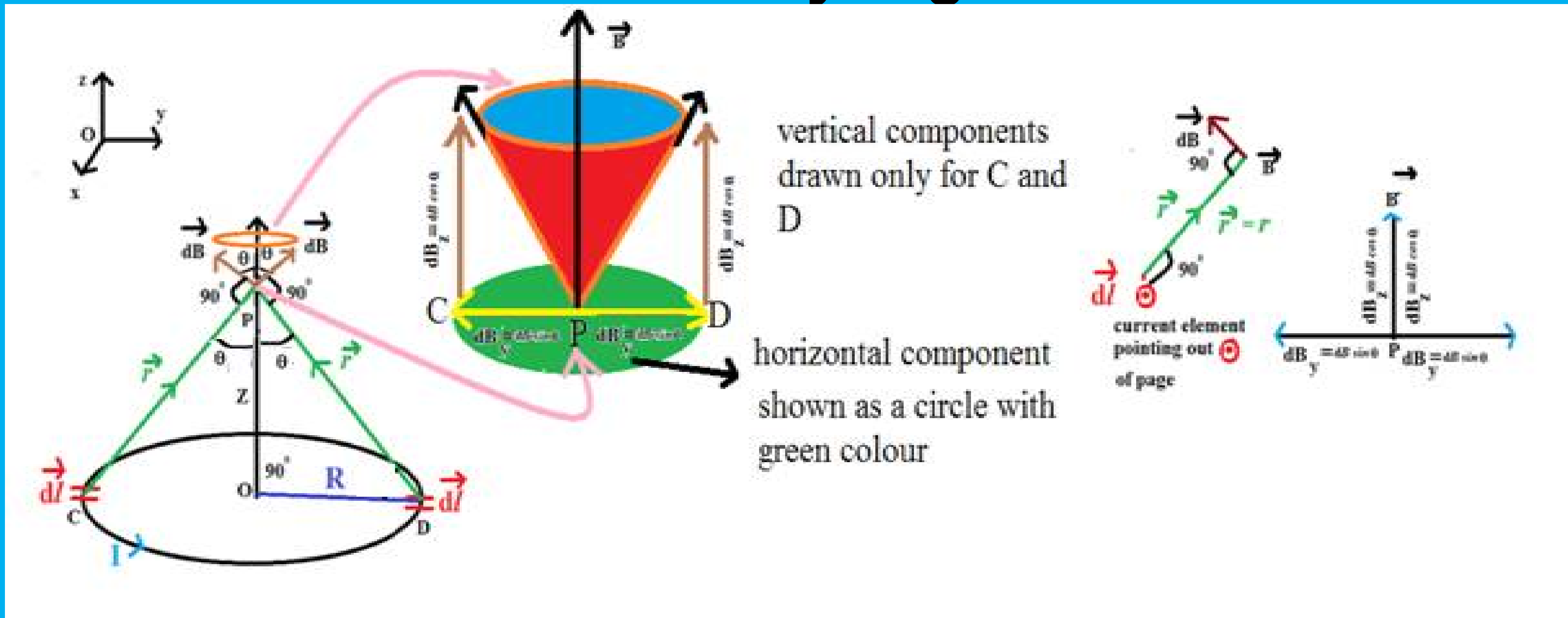
Hence,

$$\vec{B} = \frac{\mu_\circ I}{4\pi a}\left(\cos\left(\frac{\pi}{2} - \theta_1\right) - \cos\left(\frac{\pi}{2} + \theta_2\right)\right)\hat{n}$$

$$= \frac{\mu_\circ I}{4\pi a}(\sin\theta_1 + \sin\theta_2)\hat{n}$$



Magnetic field produced along the axis of the current carrying circular coil



Assume that current I flows in anti-clockwise direction

$$PC = PD = r = \sqrt{R^2 + Z^2} \; and$$

$$angle \; \angle CPO = \angle DPO = \theta$$

According to Biot-Savart's law, the magnetic field at P due to the current element $I\,d\vec{l}$ is

$$d\vec{B} = \frac{\mu_\circ}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2}$$

If we integrate $d\vec{l}$ around the loop, $d\vec{B}$ sweeps out a cone then the net magnetic field $\vec{B}$

$$\vec{\mathrm{B}} = \int d\vec{B} = \int dB\cos\theta\,\hat{k}$$

$$\vec{\mathrm{B}} = \frac{\mu_\circ I}{4\pi}\int\frac{dl}{r^2}\cos\theta\,\hat{k}$$

But $\cos\theta = \frac{R}{\left(R^2 + Z^2\right)^{\frac{1}{2}}}$, using Pythagorous theorem $r^2 = R^2 + Z^2$ and integrating line element from 0 to $2\pi R$, we get

$$\vec{B} = \frac{\mu_0 I}{2} \frac{R^2}{(R^2 + z^2)^{\frac{3}{2}}} \hat{k}$$

Note that the magnetic field $\vec{B}$ points along the direction from the point O to P. Suppose if the current flows in clockwise direction, then magnetic field points in the direction from the point P to O.

Current loop as magnetic dipole

The magnetic field from the centre of a circular loop of radius R along the axis is given by

$$\vec{B} = \frac{\mu_\circ I}{2} \frac{R^2}{\left(R^2 + z^2\right)^{\frac{3}{2}}} \hat{k}$$

At larger distance Z >> R, therefore $R^2 + Z^2 \approx Z^2$, we have

$$\vec{B} = \frac{\mu_\circ I}{2} \frac{R^2}{Z^3} \hat{k}$$

Let A be the area of the circular loop $A = \pi R^2$. So rewriting the equation in terms of area of the loop, we have

$$\vec{B} = \frac{\mu_\circ I}{2\pi}\frac{A}{Z^3}\hat{k}$$

(multiply and divide by 2)

$$\vec{B} = \frac{\mu_\circ}{4\pi}\frac{2IA}{Z^3}\hat{k}$$

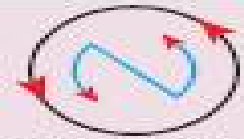
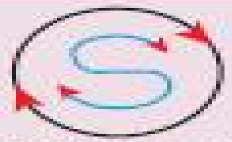
$p_m = I\,A$

where p_m is called magnetic dipole moment. In vector notation,

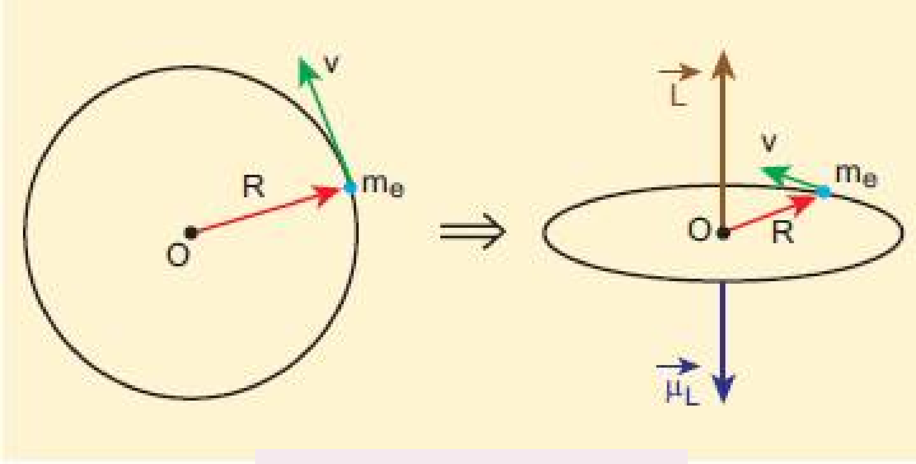
$$\vec{p}_m = I\,\vec{A}$$

This implies that a current carrying circular loop behaves as a magnetic dipole of magnetic moment . So, the magnetic dipole moment of any current loop equal to the product of the current and area of the loop.

Right hand thumb rule – In order to determine the direction of magnetic moment, we use right hand thumb rule (mnemonic) which states that **'If we curl the fingers of right hand in the direction of current in the loop, then the stretched thumb gives the direction of the magnetic moment associated with the loop'**

Current in circular loop	Polarity	Picture
Anti-clockwise current	North Pole	 Anti-clockwise current Polarity: North Pole
Clockwise current	South Pole	 Clockwise current Polarity: South Pole

Magnetic dipole moment of revolving electron



$$\vec{\mu}_L = I\vec{A}$$

In magnitude,

$$\mu_L = IA$$

If T is the time period of an electron, the current due to circular motion of the electron is

$$I = \frac{-e}{T}$$

where $-e$ is the charge of an electron. If R is the radius of the circular orbit and v is the velocity of the electron in the circular orbit, then

$$T = \frac{2\pi R}{v}$$

$$\mu_L = -\frac{e}{\frac{2\pi R}{v}}\pi R^2 = -\frac{evR}{2}$$

where A = πR^2 is the area of the circular loop. By definition, angular momentum of the electron about O is

$$\vec{L} = \vec{R} \times \vec{p}$$

In magnitude, $L = Rp = mvR$

$$\frac{\mu_L}{L} = -\frac{\frac{evR}{2}}{mvR} = -\frac{e}{2m} \Rightarrow \vec{\mu}_L = -\frac{e}{2m}\vec{L}$$

The negative sign indicates that the magnetic moment and angular momentum are in opposite direction.

In magnitude,

$$\frac{\mu_L}{L} = \frac{e}{2m} = \frac{1.60\times10^{-19}}{2\times9.11\times10^{-31}} = 0.0878\times10^{12}$$

$$\frac{\mu_L}{L} = 8.78\times10^{10}\,C\,kg^{-1} = \text{ constant}$$

The ratio $\frac{\mu_L}{L}$ is a constant and also known as gyro-magnetic ratio $\left(\frac{e}{2m}\right)$. It must be noted that the gyro-magnetic ratio is a constant of proportionality which connects angular momentum of the electron and the magnetic moment of the electron.

According to Neil's Bohr quantization rule, the angular momentum of an electron moving in a stationary orbit is quantized, which means,

$$L = n\hbar = n\frac{h}{2\pi}$$

where, h is the Planck's constant (h = 6.63 x 10^{-34} J s) and number n takes natural numbers

(i.e., n = 1,2,3,....). Hence,

$$\mu_L = n \times 9.27 \times 10^{-24} A m^2$$

The minimum magnetic moment can be obtained by substituting n = 1,

$$\mu_L = 9.27\times10^{-24}\,Am^2 = 9.27\times10^{-24}\,JT^{-1}$$
$$= (\mu_L)_{\min} = \mu_B$$

where, $\mu_B = \dfrac{eh}{4\pi m} = 9.27\times10^{-24}\,Am^2$ is called Bohr magneton. This is a convenient unit with which one can measure atomic magnetic moments.

Ampère's circuital la

Ampère's law: The line integral of magnetic field over a closed loop is μ_0 times net current enclosed by the loop.

$$\oint_C \vec{B}.\vec{dl} = \mu_o I_{enclosed}$$

where $I_{enclosed}$ is the net current linked by the closed loop C. Note that the line integral does not depend on the shape of the path or the position of the conductor with the magnetic field.

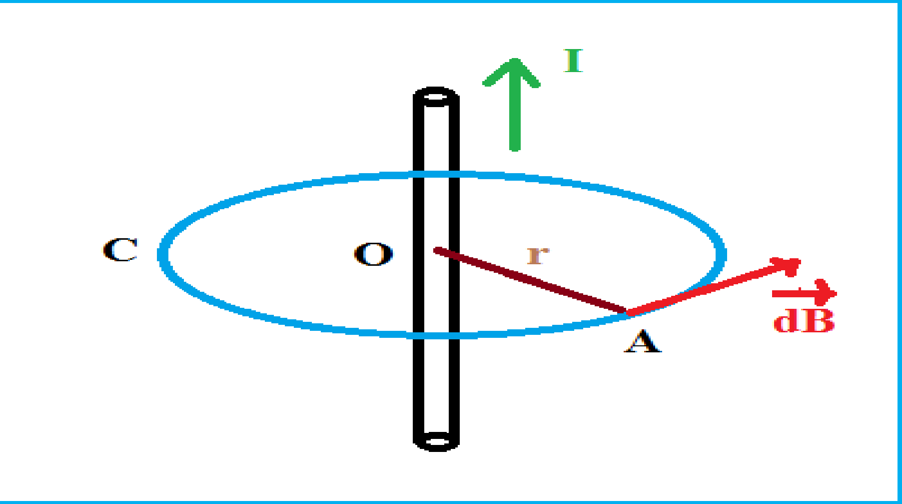
Note

Line integral means integral over a line or curve, symbol used is $\int$.

Closed line integral means integral over a closed curve (or line), symbol is $\oint$ or $\oint_c$

Applications -

- Amperian loop for current carrying straight wire



$$\oint_C \vec{B} \cdot \overrightarrow{dl} = \mu_0 I$$

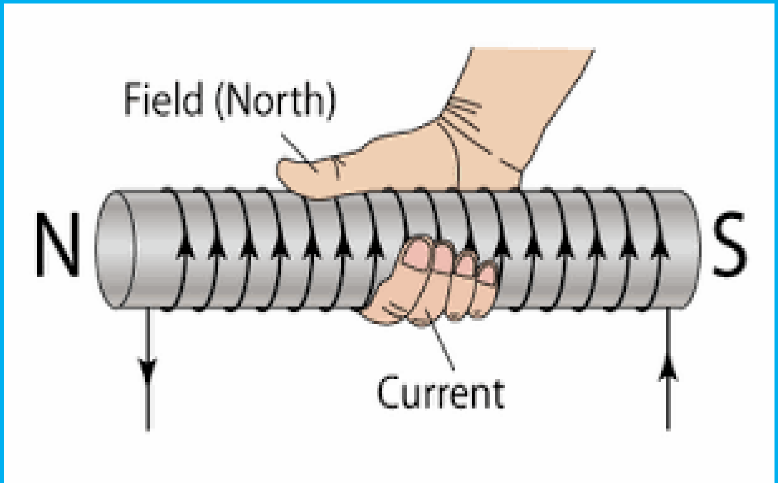
$$\oint_C B \, dl = \mu_0 I$$

$$B \int_0^{2\pi r} dl = \mu_0 I$$

$$\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{n}$$

Solenoid

- When electric current is
Passed through the solenoid
the magnetic field is Produced.
The magnetic field of the solenoid
is due to the superposition of the
magnetic fields of each turn of the solenoid. The direction
of the magnetic field of the solenoid is given by right hand
palm-rule.



Inside the solenoid

$$B = \mu_0 \frac{nLI}{L} = \mu_0 nI$$

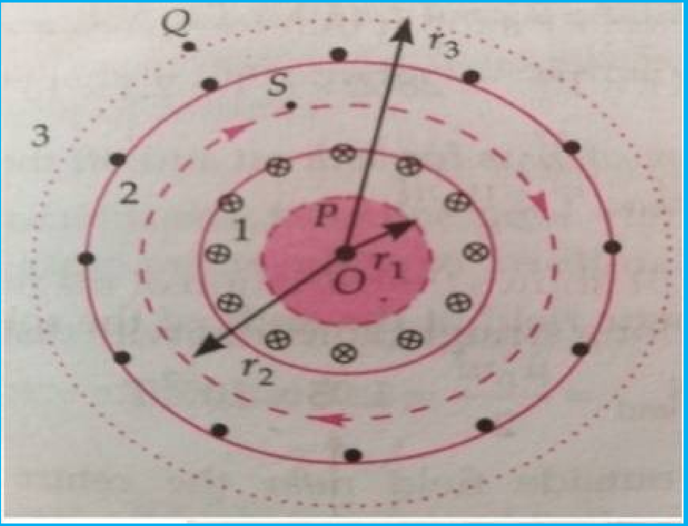
Outside the solenoid

Solenoid

Note:

Solenoid can be used as electromagnets. It produces strong magnetic field that can be turned ON or OFF. This is not possible in case of permanent magnet. Further the strength of the magnetic field can be increased by keeping iron bar inside the solenoid. This is because, the magnetic field of the solenoid magnetizes the iron bar and hence the net magnetic field is the sum of magnetic field of the solenoid and magnetic field of magnetized iron. Because of these properties, solenoids are useful in designing variety of electrical applications.

Toroid



(a) Open space interior to the toroid: $\vec{B}_P = 0$

(b) Open space exterior to the toroid: $\vec{B}_Q = 0$

(c) Inside the toroid: $B_S = \mu_0 n I$

Lorentz force

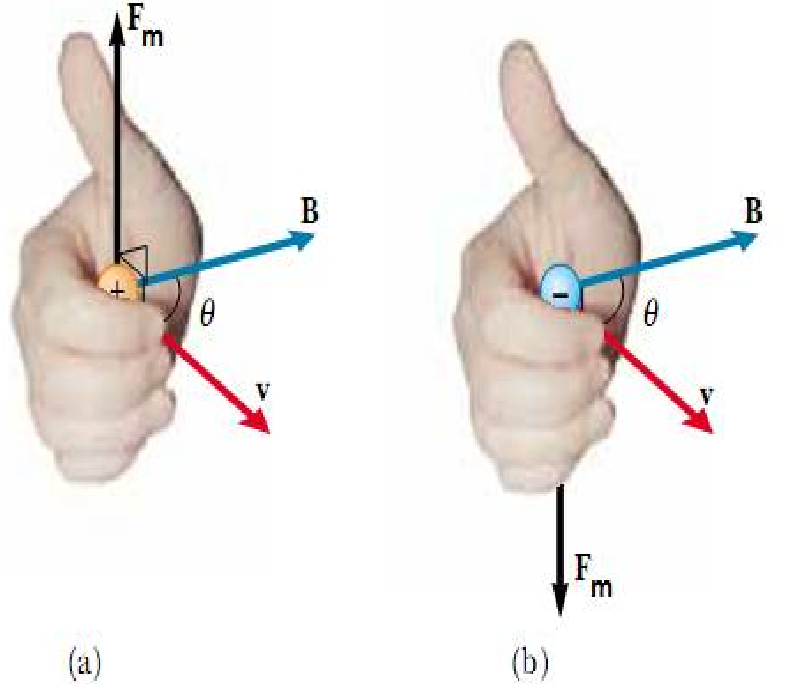
- Force experienced by a charge in electric and magnetic field

$$\vec{F}_m = q\,(\vec{v} \times \vec{B}\,)$$

- In magnitude

$$F_m = qvB\sin\theta$$

1. $\vec{F}_m$ is directly proportional to the magnetic field $\vec{B}$
2. $\vec{F}_m$ is directly proportional to the velocity $\vec{v}$
3. $\vec{F}_m$ is directly proportional to sine of the angle between the velocity and magnetic field
4. $\vec{F}_m$ is directly proportional to the magnitude of the charge q
5. The direction of $\vec{F}_m$ is always perpendicular to $\vec{v}$ and $\vec{B}$ as $\vec{F}_m$ is the cross product of $\vec{v}$ and $\vec{B}$



6. The direction of $\vec{F}_m$ on negative charge is opposite to the direction of $\vec{F}_m$ on positive charge provided other factors are identical
7. If velocity $\vec{v}$ of the charge q is along magnetic field $\vec{B}$ then, $\vec{F}_m$ is zero

Definition of tesla

The strength of the magnetic field is one tesla if unit charge moving in it with unit velocity experiences unit force.

$$1\ T = \frac{1 N s}{C m} = 1 \frac{N}{A m} = 1 N\ A^{-1} m^{-1}$$

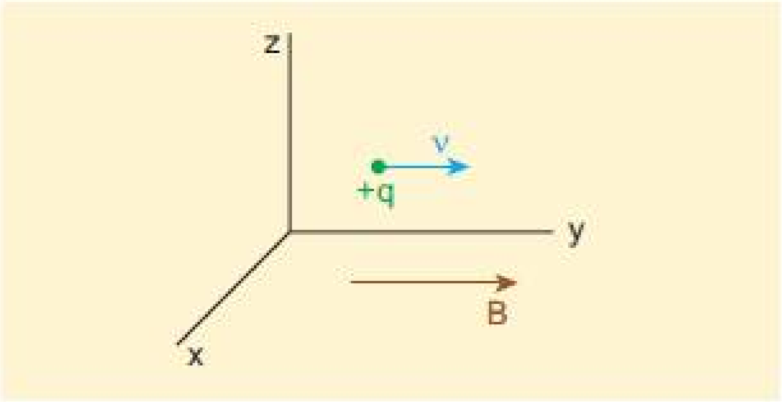
EXAMPLE 3.20

A particle of charge q moves with velocity $\vec{v}$ along positive y - direction in a magnetic field $\vec{B}$. Compute the Lorentz force experienced by the particle (a) when magnetic field is along positive y-direction (b) when magnetic field points in positive z - direction (c) when magnetic field is in zy - plane and making an angle θ with velocity of the particle. Mark the direction of magnetic force in each case.

Solution

Velocity of the particle is $\vec{v} = v\,\hat{j}$

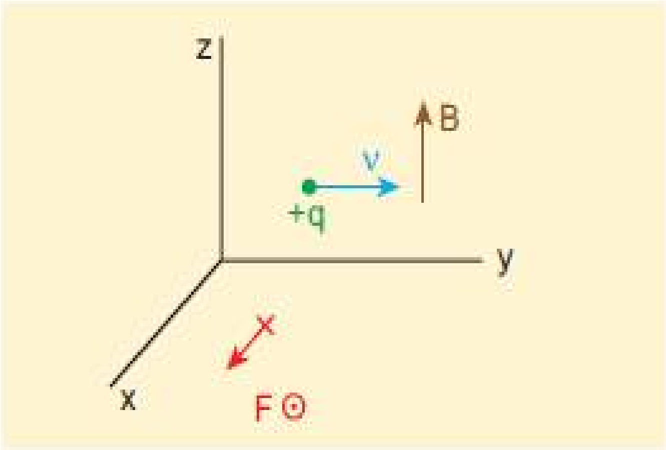
(a) Magnetic field is along positive y - direction, this implies, $\vec{B} = v\,\hat{j}$



From Lorentz force, $\vec{F}_m = q\left(v\,\hat{j} \times B\,\hat{j}\right) = 0$

So, no force acts on the particle when it moves along the direction of magnetic field.

(b) Magnetic field points in positive z - direction, this implies, $\vec{B} = B\hat{k}$

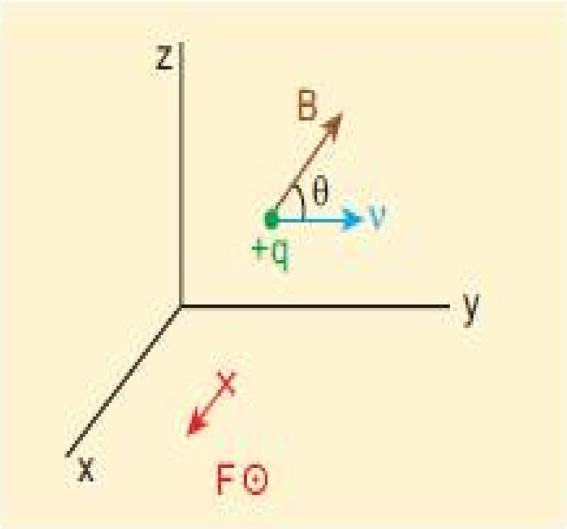


From Lorentz force,

$$\vec{F}_m = q\left(v\hat{j} \times B\hat{k}\right) = qvB\,\hat{i}$$

Therefore, the magnitude of the Lorentz force is qvB and direction is along positive x - direction.

(c) Magnetic field is in zy - plane and making an angle θ with the velocity of the particle, which implies $\vec{B} = B\cos\theta\,\hat{j} + B\sin\theta\,\hat{k}$



From Lorentz force,

$$\vec{F}_m = q\left(v\hat{j}\right) \times \left(B\cos\theta\hat{j} + B\sin\theta\,\hat{k}\right)$$

$$= qvB\sin\theta\hat{i}$$

EXAMPLE 3.21

Compute the work done and power delivered by the Lorentz force on the particle of charge q moving with velocity $\vec{v}$. Calculate the angle between Lorentz force and velocity of the charged particle and also interpret the result.

Solution

For a charged particle moving on a magnetic field, $\vec{F} = q(\vec{v} \times \vec{B})$

The work done by the magnetic field is

$$W = \int \vec{F}.d\vec{r} = \int \vec{F}.\vec{v}dt$$

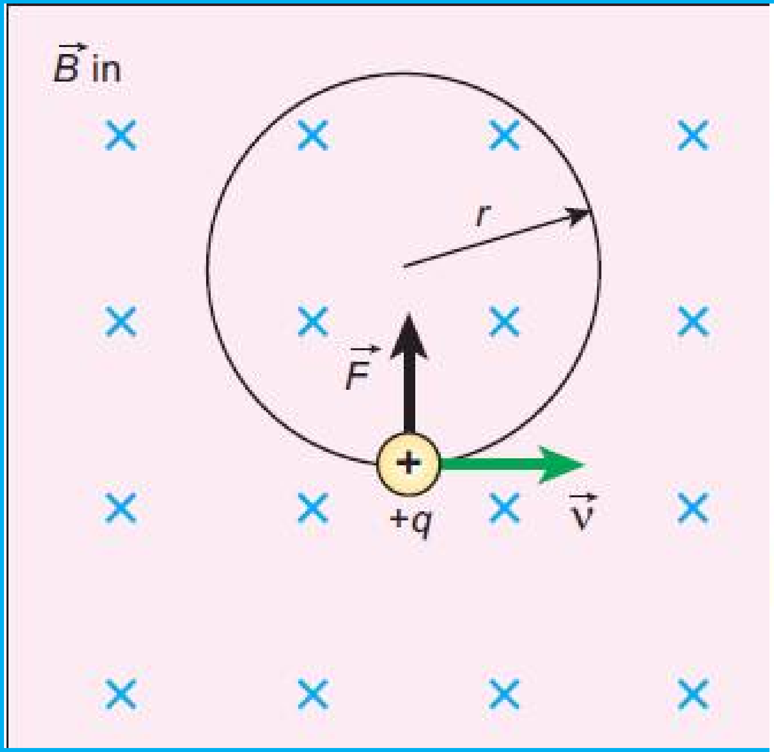
$$W = q\int (\vec{v} \times \vec{B}).\vec{v}\,dt = 0$$

Since $\vec{v} \times \vec{B}$ is perpendicular to $\vec{v}$ and hence $(\vec{v} \times \vec{B}) \cdot \vec{v} = \vec{0}$ This means that Lorentz force do no work on the particle. From work-kinetic energy theorem, (Refer section 4.2.6, XI th standard Volume I)

$$\frac{dW}{dt} = P = 0$$

Since, $\vec{F}.\vec{v} = 0 \Rightarrow \vec{F}$ and $\vec{v}$ are perpendicular to each other. The angle between Lorentz force and velocity of the charged particle is 90°. Thus Lorentz force changes the direction of the velocity but not the magnitude of the velocity. Hence Lorentz force does no work and also does not alter kinetic energy of the particle.

Motion of charged particle in a perpendicular uniform magnetic field



The Lorentz force on the charged particle is given by

$$\vec{F} = q(\vec{v} \times \vec{B})$$

Since Lorentz force alone acts on the particle, the magnitude of the net force on the particle is

$$\sum_i F_i = F_m = qvB$$

This Lorentz force acts as centripetal force for the particle to execute circular motion. Therefore,

$$qvB = m\frac{v^2}{r}$$

The radius of the circular path is

$$r = \frac{mv}{qB} = \frac{p}{qB}$$

where p = mv is the magnitude of the linear momentum of the particle. Let T be the time taken by the particle to finish one complete circular motion, then

$$T = \frac{2\pi r}{v} \Rightarrow T = \frac{2\pi m}{qB}$$

Cyclotron

- Cyclotron time period

$$T = \frac{2\pi m}{qB}$$

- Cyclotron frequency or gyro-frequency

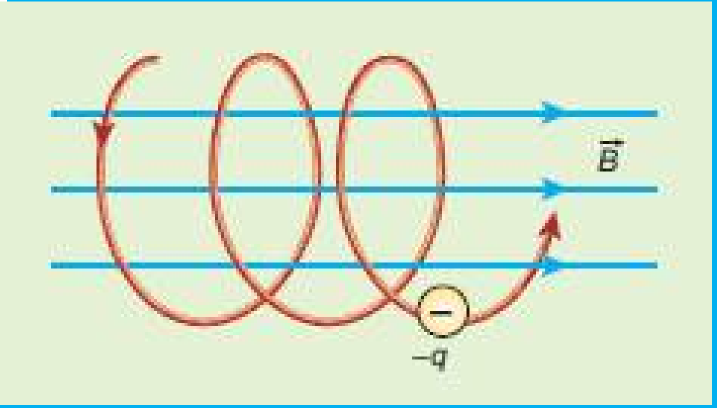
$$f = \frac{1}{T}$$

$$f = \frac{qB}{2\pi m}$$

$$\omega = 2\pi f = \frac{q}{m} B$$

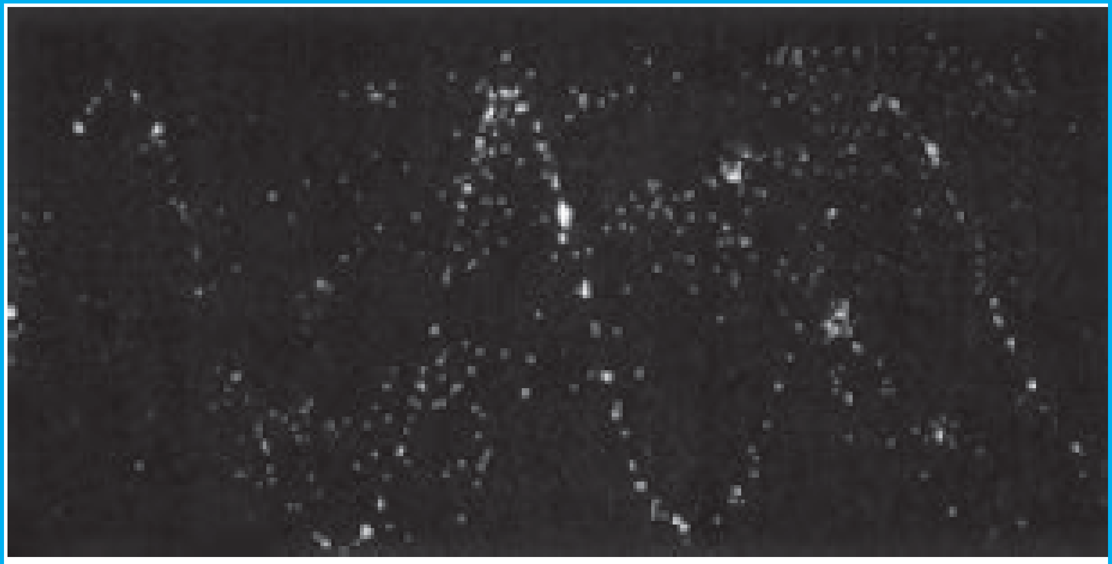
If a charged particle moves in a region of uniform magnetic field such that its velocity is not perpendicular to the magnetic field, then the velocity of the particle is split up into two components; one component is parallel to the field while the other perpendicular to the field. The component of velocity parallel to field remains unchanged and the component perpendicular to field keeps changing due to the Lorentz force. Hence the path of the particle is not a circle; it is

a helix around the field lines



Helical path of an electron

- Helical path of an electron when it moves in a magnetic field. Inside the particle detector called cloud chamber, the path is made visible by the condensation of water droplets.



Problem 3.22 – An electron moving perpendicular to a uniform magnetic field 0.500 T undergoes circular motion of radius 2.50 mm. What is the speed of electron?

Solution

Charge of an electron q = -1.60 × 10^{-19} C

$\Rightarrow |q| = 1.60 \times 10^{-19} C$

Magnitude of magnetic field B = 0.500 T

Mass of the electron, m = 9.11 × 10^{-31} kg

Radius of the orbit, r = 2.50 mm = 2.50 × 10^{-3} m

Velocity of the electron, $v = |q|\frac{rB}{m}$

Velocity of the electron, $v = |q|\frac{rB}{m}$

$$v = 1.60 \times 10^{-19} \times \frac{2.50 \times 10^{-3} \times 0.500}{9.11 \times 10^{-31}}$$

$$v = 2.195 \times 10^{8} ms^{-1}$$

EXAMPLE 3.23

A proton moves in a uniform magnetic field of strength 0.500 T magnetic field is directed along the x-axis. At initial time, $t = 0\ s$, the proton has velocity $\vec{v} = \left(1.95\times10^{5}\hat{i} + 2.00\times10^{5}\hat{k}\right) m\ s^{-1}$. Find

(a) At initial time, what is the acceleration of the proton.

(b) Is the path circular or helical?. If helical, calculate the radius of helical trajectory and also calculate the pitch of the helix (Note: Pitch of the helix is the distance travelled along the helix axis per revolution).

Solution

Magnetic field $\vec{B} = 0.500\hat{i}\ T$

Velocity of the particle

$$\vec{v} = \left(1.95\times10^{5}\hat{i} + 2.00\times10^{5}\hat{k}\right) m s^{-1}$$

Charge of the proton $q = 1.60\times10^{-19} C$

Mass of the proton $m = 1.67\times10^{-27} kg$

(a) The force experienced by the proton is

$$\vec{F} = q\left(\vec{v}\times\vec{B}\right)$$

$$= 1.60\times10^{-19}\times\left(\left(1.95\times10^{5}\hat{i} + 2.00\times10^{5}\hat{k}\right)\times\left(0.500\hat{i}\right)\right)$$

$$\vec{F} = 1.60\times10^{-14} N\,\hat{j}$$

Therefore, from Newton's second law,

$$\vec{a} = \frac{1}{m}\vec{F} = \frac{1}{1.67\times10^{-27}}\left(1.60\times10^{-14}\right)$$

$$= 9.58\times10^{12}\, m\,s^{-2}$$

(b) Trajectory is helical

Radius of helical path is

$$R = \frac{mv_z}{|q|B} = \frac{1.67\times10^{-27}\times2.00\times10^{5}}{1.60\times10^{-19}\times0.500}$$

$$= 4.175\times10^{-3} m = 4.18 mm$$

Pitch of the helix is the distance travelled along x-axis in a time T, which is $P = v_x T$

But time,

$$T=\frac{2\pi}{\omega}=\frac{2\pi m}{|q|B}=\frac{2\times3.14\times1.67\times10^{-27}}{1.60\times10^{-19}\times0.500}$$

$$=13.1\times10^{-8}s$$

Hence, pitch of the helix is

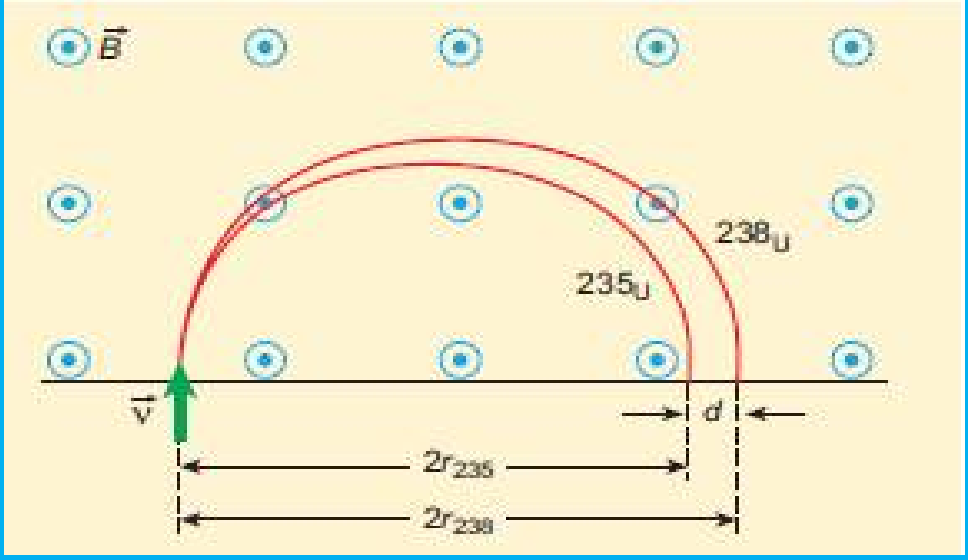
$$P=v_xT=\left(1.95\times10^{5}\right)\left(13.1\times10^{-8}\right)$$

$$=25.5\times10^{-3}m=25.5\ mm$$

The proton experiences appreciable acceleration in the magnetic field, hence the pitch of the helix is almost six times greater than the radius of the helix.

EXAMPLE 3.24

Two singly ionized isotopes of uranium $^{235}_{92}U$ and $^{238}_{92}U$ (isotopes have same atomic number but different mass number) are sent with velocity 1.00×10^5 m s^{-1} into a magnetic field of strength 0.500 T normally. Compute the distance between the two isotopes after they complete a semi-circle. Also compute the time taken by each isotope to complete one semi-circular path. (Given: masses of the isotopes: m_{235} = 3.90×10^{-25} kg and m_{238} = 3.95×10^{-25} kg)



Solution

Since isotopes are singly ionized, they have equal charge which is equal to the charge of an electron, q = - 1.6 × 10^{-19} C. Mass of uranium ${}^{235}_{92}U$ and ${}^{238}_{92}U$ are 3.90 × 10^{-25} kg and 3.95 × 10^{-25} kg respectively. Magnetic field applied, B = 0.500 T. Velocity of the electron is 1.00 × 10^{5} m s^{-1}, then

(a) the radius of the path of ${}^{235}_{92}U$ is r_{235}

$$r_{235} = \frac{m_{235}v}{|q|B} = \frac{3.90\times10^{-25}\times1.00\times10^{5}}{1.6\times10^{-19}\times0.500}$$

$$= 48.8\times10^{-2}\,m$$

$$r_{235} = 48.8 \text{ cm}$$

The diameter of the semi-circle due to ${}^{235}_{92}U$ is $d_{235} = 2r_{235} = 97.6$ cm

The radius of the path of ${}^{238}_{92}U$ is r_{238} then

$$r_{238} = \frac{m_{238}v}{|q|B} = \frac{3.95\times10^{-25}\times1.00\times10^{5}}{1.6\times10^{-19}\times0.500} = 49.4\times10^{-2}\,m$$

The diameter of the semi-circle due to ${}^{238}_{92}U$ is $d_{238} = 2r_{238} = 98.8$ cm

Therefore the separation distance between the isotopes is $\Delta d = d_{238} - d_{235} = 1.2\,\text{cm}$

(b) The time taken by each isotope to complete one semi-circular path are

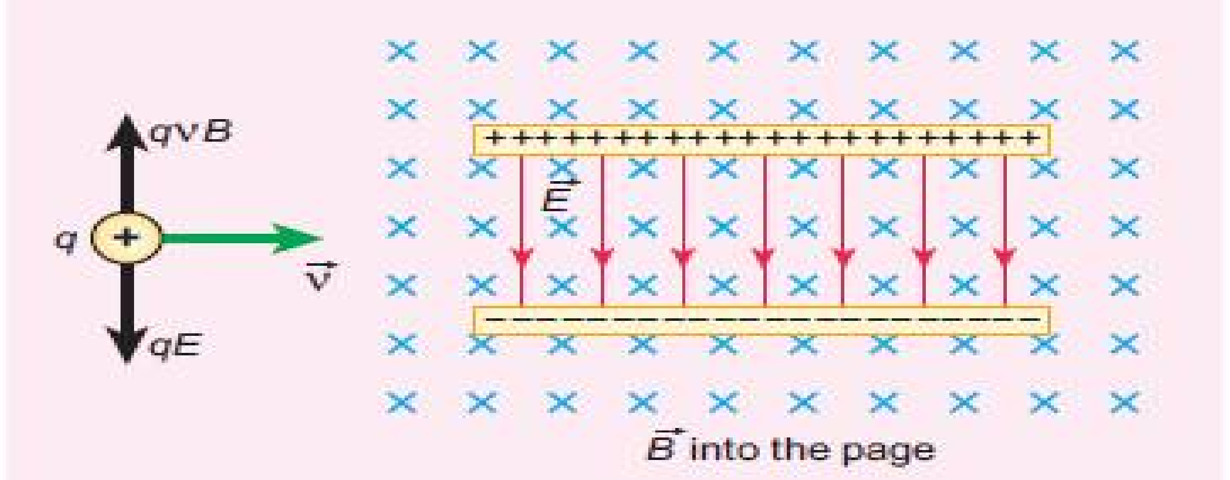
$$t_{235} = \frac{\text{magnitude of the displacement}}{\text{velocity}}$$

$$= \frac{97.6\times10^{-2}}{1.00\times10^{5}} = 9.76\times10^{-6}\,s = 9.76\,\mu s$$

$$t_{238} = \frac{\text{magnitude of the displacement}}{\text{velocity}}$$

$$= \frac{98.8\times10^{-2}}{1.00\times10^{5}} = 9.88\times10^{-6}\,s = 9.88\,\mu s$$

Velocity selector



$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$$qE = qv_{\circ}B$$

$$\Longrightarrow v_{\circ} = \frac{E}{B}$$

This means, for a given magnitude of $\vec{E}$- field and $\vec{B}$- field, the forces act only for the particle moving with particular speed $v_{\circ} = \frac{E}{B}$. This speed is independent of mass and charge.

S.No.	Velocity	Deflection
1	$v > v_o$	Charged particle deflects in the direction of Lorentz force
2	$v < v_o$	Charged particle deflects in the direction of Coulomb force
3	$v = v_o$	No deflection and particle moves straight

So by proper choice of electric and magnetic fields, the particle with particular speed can be selected. Such an arrangement of fields is called a **velocity selector.**

This principle is used in Bainbridge mass spectrograph to separate the isotopes.

EXAMPLE 3.25

Let E be the electric field of magnitude 6.0×10^6 N C^{-1} and B be the magnetic field magnitude 0.83 T. Suppose an electron is accelerated with a potential of 200 V, will it show zero deflection?. If not, at what potential will it show zero deflection.

Solution:

Electric field, E = 6.0×10^6 N C^{-1} and magnetic field, B = 0.83 T.

Then

$$v = \frac{E}{B} = \frac{6.0 \times 10^6}{0.83} = 7.23 \times 10^6\, m s^{-1}$$

When an electron goes with this velocity, it shows null deflection. Since the accelerating potential is 200 V, the electron acquires kinetic energy because of this accelerating potential. Hence,

$$\frac{1}{2} m v^2 = eV \Rightarrow v = \sqrt{\frac{eV}{2m}}$$

Since the mass of the electron, $m = 9.1 \times 10^{-31}\, kg$ and charge of an electron, $|q| = e = 1.6 \times 10^{-19}\, C$. The velocity due to accelerating potential 200 V

$$v_{200} = \sqrt{\frac{2\left(1.6 \times 10^{-19}\right)(200)}{\left(9.1 \times 10^{-31}\right)}} = 8.39 \times 10^6\, m\, s^{-1}$$

Since the speed $v_{200} > v$, the electron is deflected towards direction of Lorentz force. So, in order to have null deflection, the potential, we have to supply is

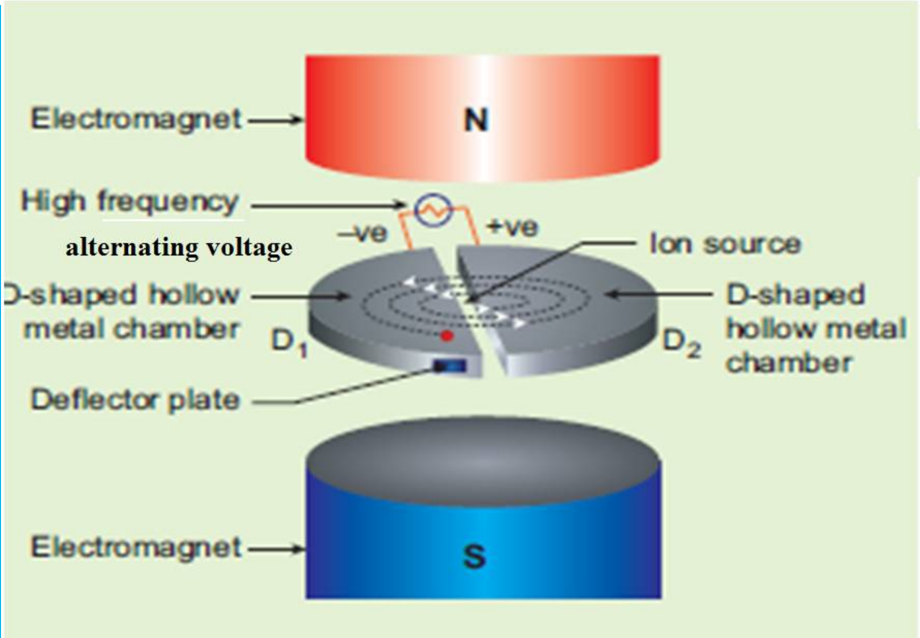
$$V = \frac{1}{2} \frac{mv^2}{e} = \frac{\left(9.1 \times 10^{-31}\right) \times \left(7.23 \times 10^6\right)^2}{2 \times \left(1.6 \times 10^{-19}\right)}$$

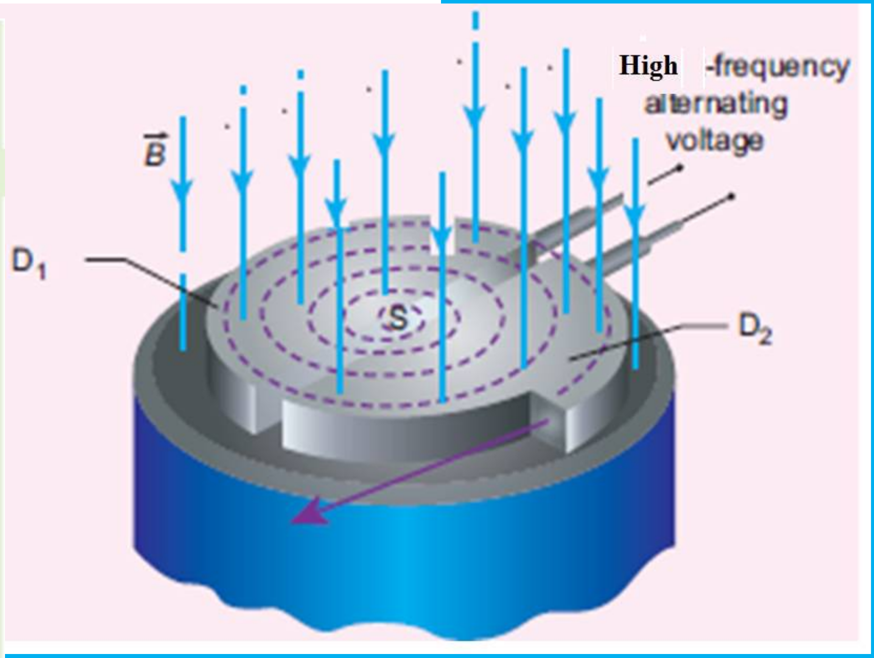
$$V = 148.65\, V$$

Cyclotron

Principle

When a charged particle moves normal to the magnetic field, it experiences magnetic Lorentz force.





$$\frac{mv^2}{r} = qvB$$

$$\Rightarrow r = \frac{m}{qB} v$$

$$\Rightarrow r \propto v$$

$$f_{OSC} = \frac{qB}{2\pi m}$$

The time period of oscillation is

$$T = \frac{2\pi m}{qB}$$

The kinetic energy of the charged particle is

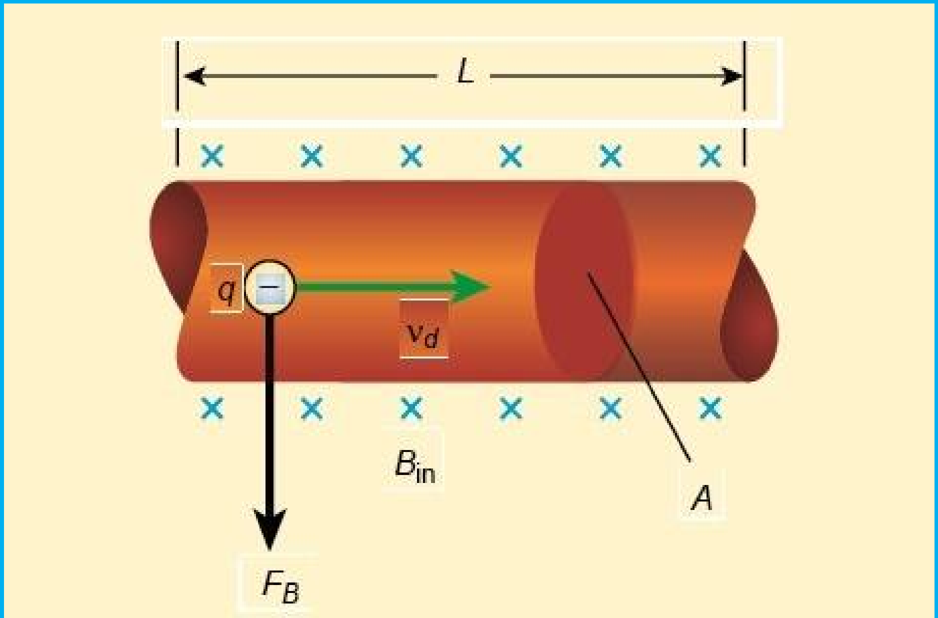
$$KE = \frac{1}{2} mv^2 = \frac{q^2 B^2 r^2}{2m}$$

Very important condition in cyclotron operation is the resonance condition. It happens when the frequency f at which the positive ion circulates in the magnetic field must be equal to the constant frequency of the electrical oscillator f_{osc}

Limitations of cyclotron

(a) the speed of the ion is limited
(b) electron cannot be accelerated
(c) uncharged particles cannot be accelerated

Force on a current carrying conductor placed in a magnetic field



Let n be the number of free electrons per unit volume, therefore

$$n = \frac{N}{V}$$

where N is the number of free electrons in the small element of volume V = A *dl*.

Hence Lorentz force on the wire of length d*l* is the product of the number of the electrons

(N = nA d*l*) and the force acting on an electron.

$$d\vec{F} = -enAdl\left(\vec{v}_d \times \vec{B}\right)$$

$$I = neAv_d$$

$$\vec{F} = -e\left(\vec{v}_d \times \vec{B}\right)$$

The length dl is along the length of the wire and hence the current element in the wire is $I\,d\vec{l} = -enA\vec{v}_d dl$ Therefore the force on the wire is

$$d\vec{F} = \left(I\,d\vec{l} \times \vec{B}\right)$$

The force in a straight current carrying conducting wire of length l placed in a uniform magnetic field is

$$\vec{F} = \left(I\vec{l} \times \vec{B}\right)$$

In magnitude,

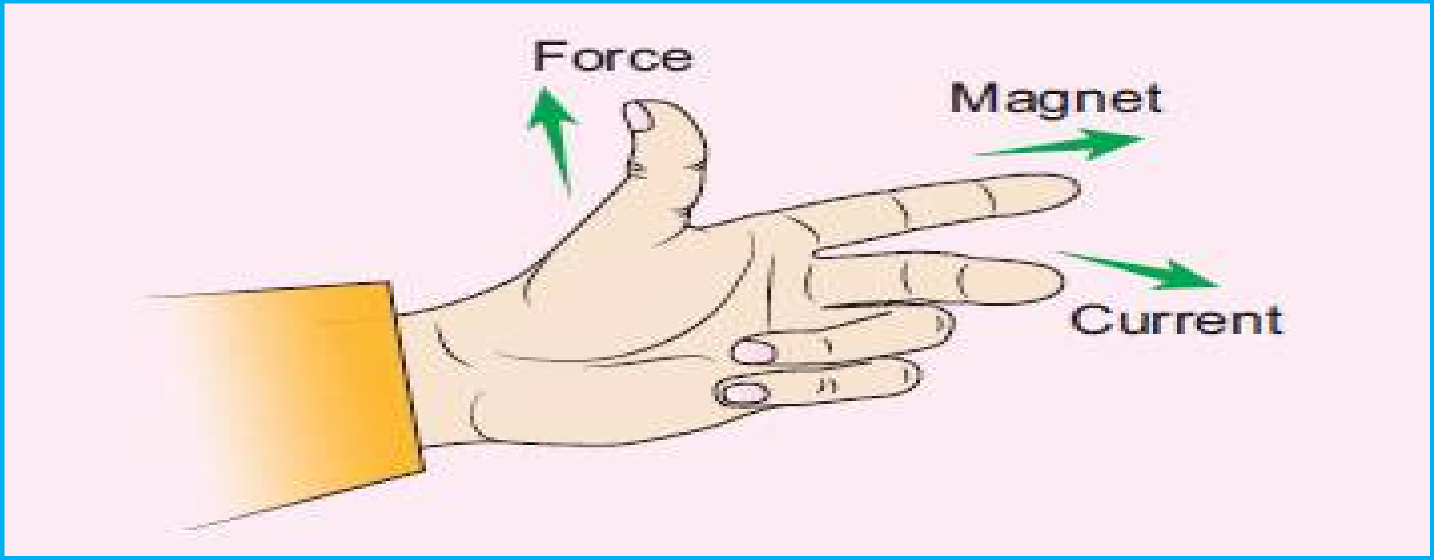
$$F = BIl\sin\theta$$

(a) If the conductor is placed along the direction of the magnetic field, the angle between them is $\theta = 0°$. Hence, the force experienced by the conductor is zero.

(b) If the conductor is placed perpendicular to the magnetic field, the angle between them is $\theta = 90°$ Hence, the force experienced by the conductor is maximum, which is $F = BIl$.

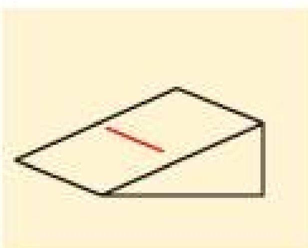
Fleming’s left hand rule (mnemonic)

- When a current carrying conductor is placed in a magnetic field, the direction of the force experienced by it is given by Fleming’s Left Hand Rule (FLHR)
- Stretch forefinger, the middle finger and the thumb of the left hand such that they are in mutually perpendicular directions. If forefinger points the direction of magnetic field, the middle finger points the direction of the electric current, then thumb will point the direction of the force experienced by the conductor.



EXAMPLE 3.27

A metallic rod of linear density is 0.25 kg m^{-1} is lying horizontally on a smooth inclined plane which makes an angle of 45° with the horizontal. The rod is not allowed to slide down by flowing a current through it when a magnetic field of strength 0.25 T is acting on it in the vertical direction. Calculate the electric current flowing in the rod to keep it stationary.

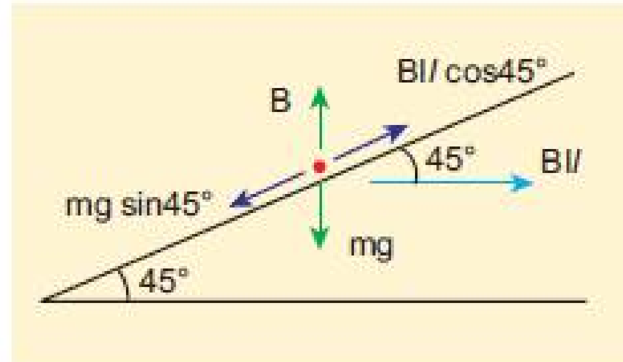


Solution

The linear density of the rod i.e., mass per unit length of the rod is 0.25 kg m^{-1}

$$\Rightarrow \frac{m}{l} = 0.25 \text{ kg m}^{-1}$$

Let I be the current flowing in the metallic rod. The direction of electric current is into the paper. The direction of magnetic force IBl is given by Fleming's left hand rule.



For equilibrium,

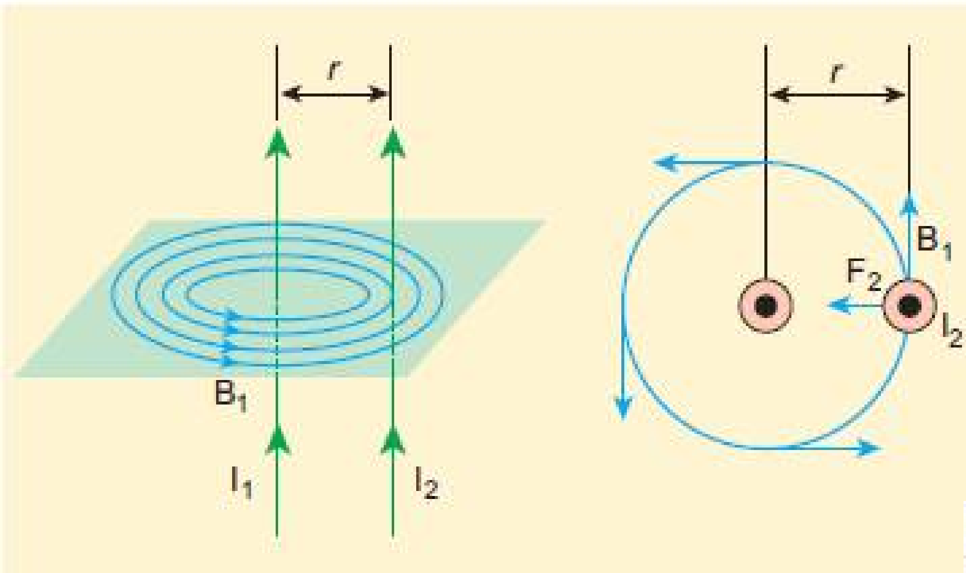
$$mg\sin 45^\circ = IBl\cos 45^\circ$$

$$\Rightarrow I = \frac{1}{B}\frac{m}{l} g \tan 45^\circ$$

$$= \frac{0.25 \text{ kg m}^{-1}}{0.25 T} \times 1 \times 9.8 \, m\,s^{-2}$$

$$\Rightarrow I = 9.8 \, A$$

Force between two long parallel current carrying conductors



Let I_1 and I_2 be the electric currents passing through the conductors A and B in same direction (i.e. along z - direction) respectively. The net magnetic field at a distance r due to current I_1 in conductor A is

$$\vec{B}_1 = \frac{\mu_\circ I_1}{2\pi r}(-\hat{i}) = -\frac{\mu_\circ I_1}{2\pi r}\hat{i}$$

From thumb rule, the direction of magnetic field is perpendicular to the plane of the paper and inwards (arrow into the page ⊗) i.e. along negative $\hat{i}$ direction.

Let us consider a small elemental length dl in conductor B at which the magnetic field $\vec{B}_1$ is present.

$$d\vec{F} = \left(I_2 d\vec{l} \times \vec{B}_1\right) = -I_2 dl \frac{\mu_\circ I_1}{2\pi r}(\hat{k} \times \hat{i})$$

$$= -\frac{\mu_\circ I_1 I_2 dl}{2\pi r}\hat{j}$$

Therefore the force on dl of the wire B is directed towards the wire W_1. So the length dl is attracted towards the conductor A. The force per unit length of the conductor B due to the wire conductor A is

$$\frac{\vec{F}}{l} = -\frac{\mu_o I_1 I_2}{2\pi r}\hat{j}$$

In the same manner, we compute the magnitude of net magnetic induction due to current I_2 (in conductor A) at a distance r in the elemental length dl of conductor A is

$$\vec{B}_2 = \frac{\mu_o I_2}{2\pi r}\hat{i}$$

From the thumb rule, direction of magnetic field is perpendicular to the plane of the paper and outwards (arrow out of the page ⊙) i.e., along positive $\hat{i}$ direction.

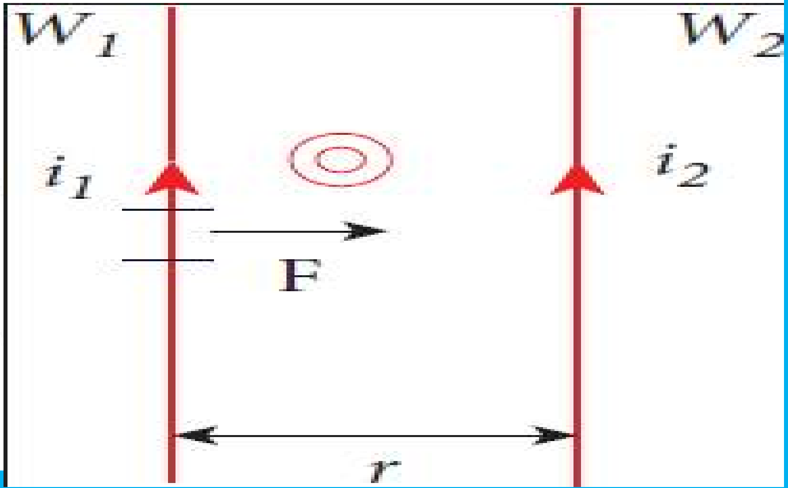
Hence, the magnetic force at element dl of the wire is W_1 is

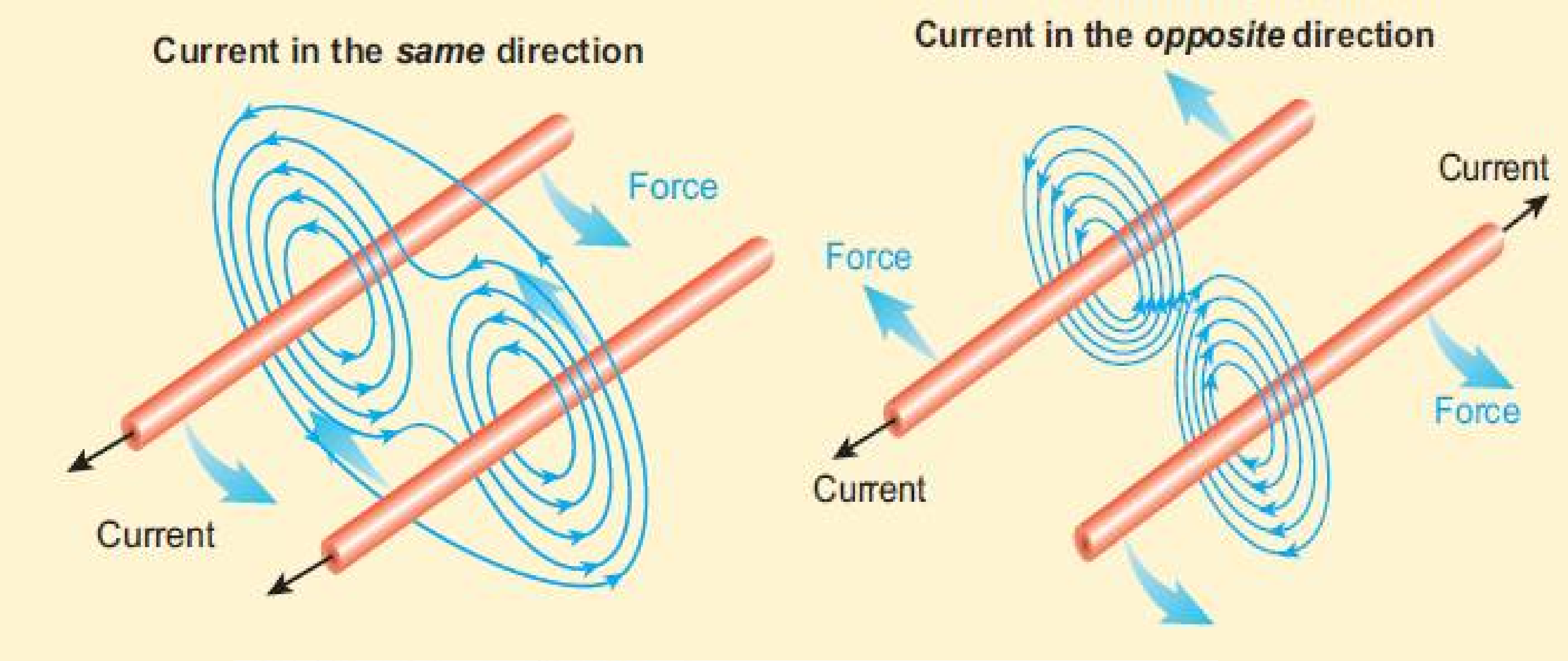
$$\vec{F} = \left(I_1\vec{d}l \times \vec{B}_2\right) = I_1 dl\frac{\mu_o I_2}{2\pi r}(\hat{k} \times \hat{i})$$

$$= \frac{\mu_o I_1 I_2 dl}{2\pi r}\hat{j}$$

The force per unit length of the conductor A due to the conductor B is

$$\frac{\vec{F}}{l} = -\frac{\mu_o I_1 I_2}{2\pi r}\hat{j}$$



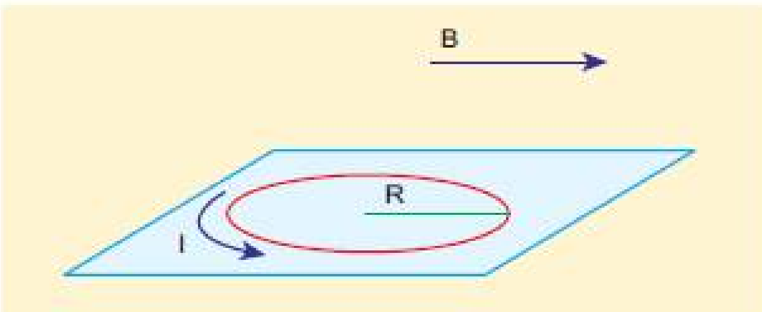


Definition of ampère

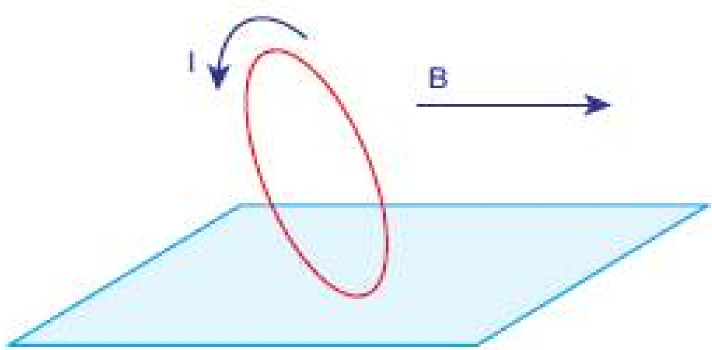
One ampère is defined as that current when it is passed through each of the two infinitely long parallel straight conductors kept at a distance of one meter apart in vacuum causes each conductor to experience a force of 2×10^{-7} newton per meter length of conductor.

EXAMPLE 3.28

Consider a circular wire loop of radius R, mass m kept at rest on a rough surface. Let I be the current flowing through the loop and $\vec{B}$ be the magnetic field acting along horizontal as shown in Figure. Estimate the current I that should be applied so that one edge of the loop is lifted off the surface?



Solution



When the current is passed through the loop, the torque is produced. If the torque acting on the loop is increased then the loop will start to rotate. The loop will start to lift if and only if the magnitude of magnetic torque due to current applied equals to the gravitational torque as shown in Figure

$$\tau_{magnetic} = \tau_{gravitational}$$

$$IAB = mgR$$

But $p_m = IA = I\left(\pi R^2\right)$

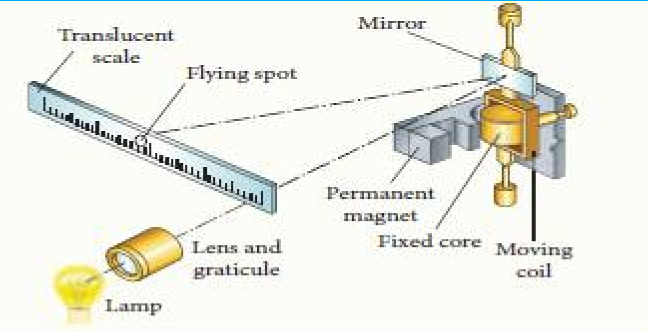
$$\pi I R^2 B = mgR$$

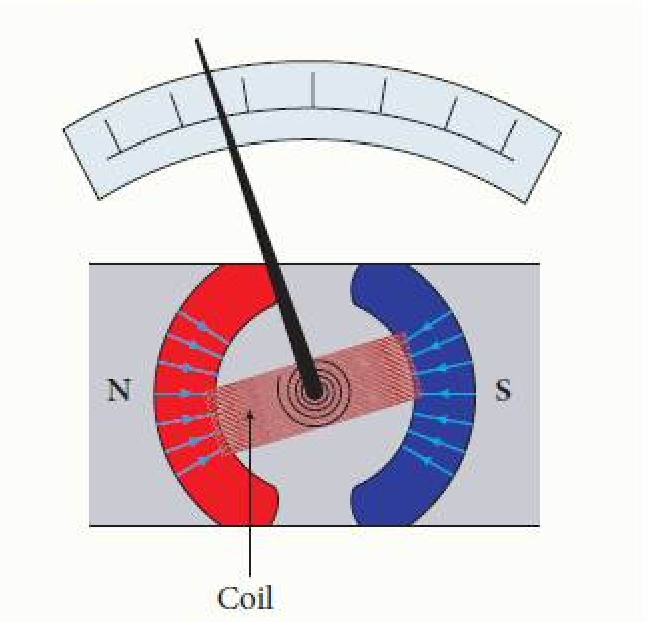
$$\Rightarrow I = \frac{mg}{\pi R B}$$

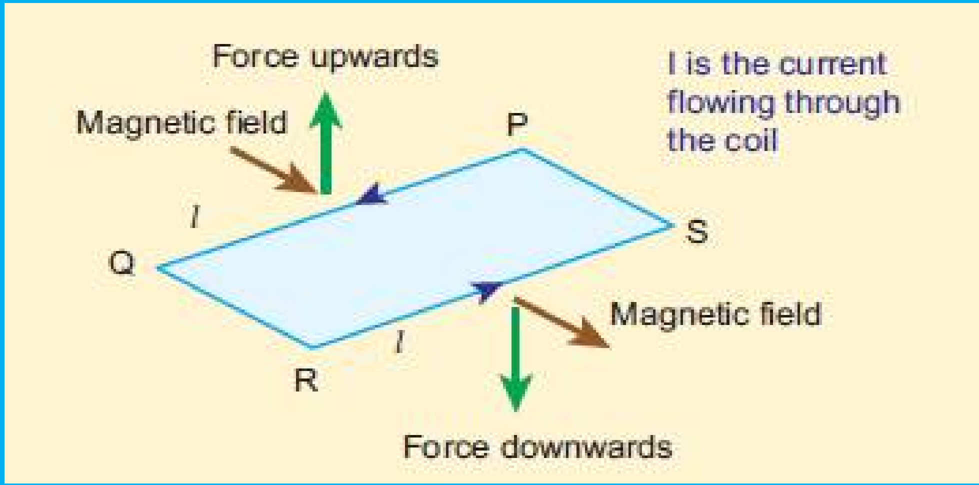
The current estimated using this equation should be applied so that one edge of loop is lifted of the surface.

Moving coil galvanometer

- Moving coil galvanometer is a device which is used to indicate the flow of current in an electrical circuit.
- Principle : When a current carrying loop is placed in a uniform magnetic field it experiences a torque.







For coil with N turns, we get

$$\tau = NABI$$

Due to this deflecting torque, the coil gets twisted and restoring torque (also known as restoring couple) is developed. Hence the magnitude of restoring couple is proportional to the amount of twist θ (Refer Unit 10 of Std. XI Physics).Thus

$$\tau = K\theta$$

where K is the restoring couple per unit twist or torsional constant of the spring.

At equilibrium, the deflection couple is equal to the restoring couple.

$$NABI = K\theta$$

$$\Rightarrow I = \frac{K}{NAB}\theta$$

(or) $I = G\theta$

where, $G = \frac{K}{NAB}$ is called galvanometer constant or current reduction factor of the galvanometer.

Figure of merit of a galvanometer

It is defined as the current which produces a deflection of one scale division in the galvanometer.

Sensitivity of a galvanometer

The galvanometer is said to be sensitive if it shows large scale deflection even though a small current is passed through it or a small voltage is applied across it.

Current sensitivity: *It is defined as the deflection produced per unit current flowing through it.*

$$I_S = \frac{\theta}{I} = \frac{NAB}{K} \Rightarrow I_S = \frac{1}{G}$$

The current sensitivity of a galvanometer can be increased

(a) by increasing

(1) the number of turns N

(2) the magnetic induction B

(3) the area of the coil A

(b) by decreasing

the couple per unit twist of the suspension wire k. Phosphor - bronze wire is used as the suspension wire because the couple per unit twist is very small.

Voltage sensitivity: ***It is defined as the deflection produced per unit voltage applied across it.***

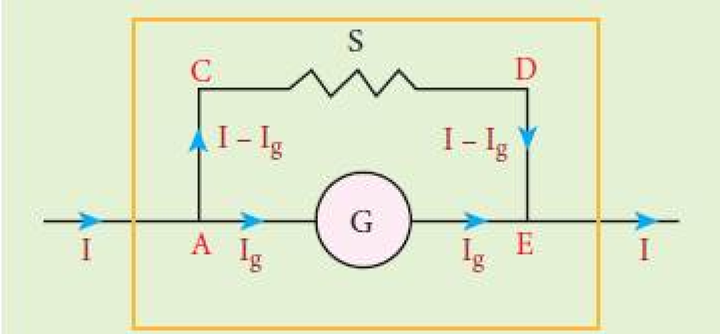
$$V_s = \frac{\theta}{V}$$

$$V_S = \frac{\theta}{IR_g} = \frac{NAB}{K\,R_g} \Rightarrow V_S = \frac{1}{GR_g} = \frac{I_S}{R_g}$$

where R_g is the resistance of galvanometer.

Conversion of galvanometer into ammeter and voltmeter

- Galvanometer to Ammeter - Shunt (low resistance) connected in parallel



Ammeter

$$V_{galvanometer} = V_{shunt}$$

$$\Rightarrow I_g R_g = (I - I_g) S$$

$$S = \frac{I_g}{(I - I_g)} R_g \text{ Or}$$

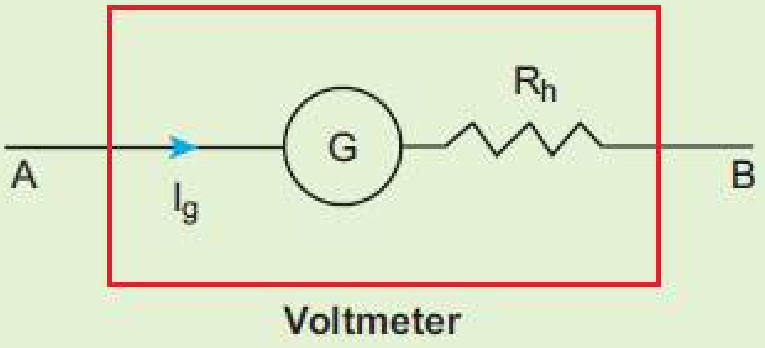
$$I_g = \frac{S}{S + R_g} I \Rightarrow I_g \propto I$$

$$\theta = \frac{1}{G} I_g \Rightarrow \theta \propto I_g \Rightarrow \theta \propto I$$

Key points

1. An ammeter is a low resistance instrument and it is always connected in series to the circuit
2. An ideal ammeter has zero resistance
3. In order to increase the range of an ammeter n times, the value of shunt resistance to be connected in parallel is $S = \frac{G}{n-1}$

Galvanometer to a voltmeter



$$I = I_g$$

$$I = I_g \Rightarrow I_g = \frac{\text{potential difference}}{\text{total resistance}}$$

Therefore,

$$I_g = \frac{V}{R_g + R_h}$$

$$\Rightarrow R_h = \frac{V}{I_g} - R_g$$

Note that $I_g \propto V$

Key points

1. Voltmeter is a high resistance instrument and it is always connected in parallel with the circuit element across which the potential difference is to be calculated
2. An ideal voltmeter has infinite resistance
3. In order to increase the range of voltmeter n times the value of resistance to be connected in series with galvanometer is R = (n-1) G

Expression for torque on a current loop placed in a magnetic field

- Consider a single rectangular loop PQRS kept in a uniform magnetic field $\vec{B}$
- Let a and b be the length and breadth of the rectangular loop respectively. The area of the loop PQRS is $A = ab$
- Let $\hat{n}$ be the unit vector normal to the plane of the current loop.
- When an electric current is passed through the loop, the net force acting is zero but there will be net torque.

(a) When unit vector $\hat{n}$ is perpendicular to the magnetic field $\vec{B}$

(a) Force on section PQ

For section PQ, $\vec{l} = -a\,\hat{j}$ and $\vec{B} = B\,\hat{i}$

$$\vec{F}_{PQ} = I\vec{l} \times \vec{B}$$
$$= -I a B(\hat{j} \times \hat{i}) = I a B\,\hat{k}$$

Since the unit vector normal to the plane $\hat{n}$ is along the direction of $\hat{k}$.

(b) The force on section QR

$\vec{l} = b\vec{i}$ and $\vec{B} = B\,\hat{i}$

$$\vec{F}_{QR} = I\vec{l} \times \vec{B} = I\,b\,B(\hat{i} \times \hat{i}) = \vec{0}$$

(c) The force on section RS

$\vec{l} = a\,\hat{j}$ and $\vec{B} = B\,\hat{i}$

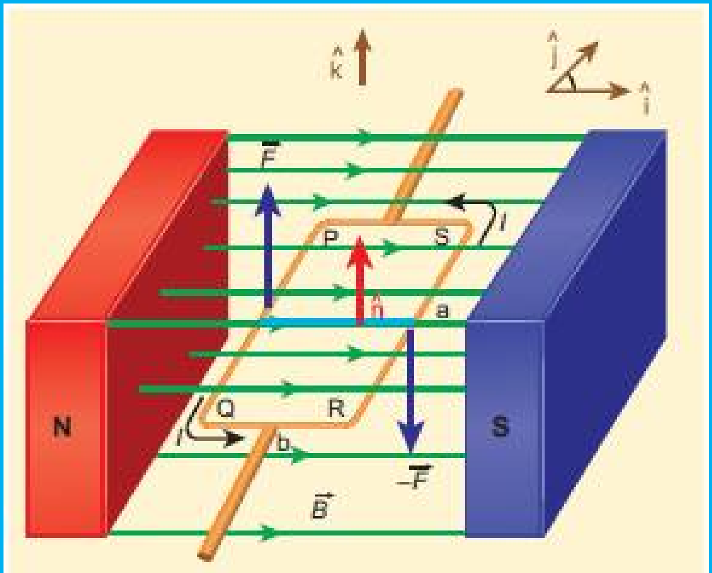
$$\vec{F}_{RS} = I\vec{l} \times \vec{B} = I\,a\,B(\hat{j} \times \hat{i}) = -IaB\,\hat{k}$$

Since, the unit vector normal to the plane is along the direction of $-\hat{k}$.

(d) The force on section SP

$\vec{l} = -b\,\hat{i}$ and $\vec{B} = B\,\hat{i}$

$$\vec{F}_{SP} = I\vec{l} \times \vec{B} = -I\,b\,B(\hat{i} \times \hat{i}) = \vec{0}$$



The net force on the rectangular loop is

$$\vec{F}_{net} = \vec{F}_{PQ} + \vec{F}_{QR} + \vec{F}_{RS} + \vec{F}_{SP}$$
$$\vec{F}_{net} = IaB\,\hat{k} + \vec{0} - IaB\,\hat{k} + \vec{0} \Rightarrow \vec{F}_{net} = \vec{0}$$

$$\vec{\tau}_{net} = \sum_{i=1}^{4} \vec{\tau}_i = \sum_{i=1}^{4} \vec{r}_i \times \vec{F}_i$$
$$= \left(\frac{b}{2} IaB + 0 + \frac{b}{2} IaB + 0\right)\hat{j}$$
$$\vec{\tau}_{net} = abIB\,\hat{j} \quad \Rightarrow \quad \vec{\tau}_{net} = ABI\,\hat{j}$$

(a) The force on section PQ

$$\vec{l} = -a\,\hat{j} \ and\ \vec{B} = B\,\hat{i}$$

$$\vec{F}_{PQ} = I\vec{l} \times \vec{B} = -IaB\left(\hat{j} \times \hat{i}\right) = IaB\,\hat{k}$$

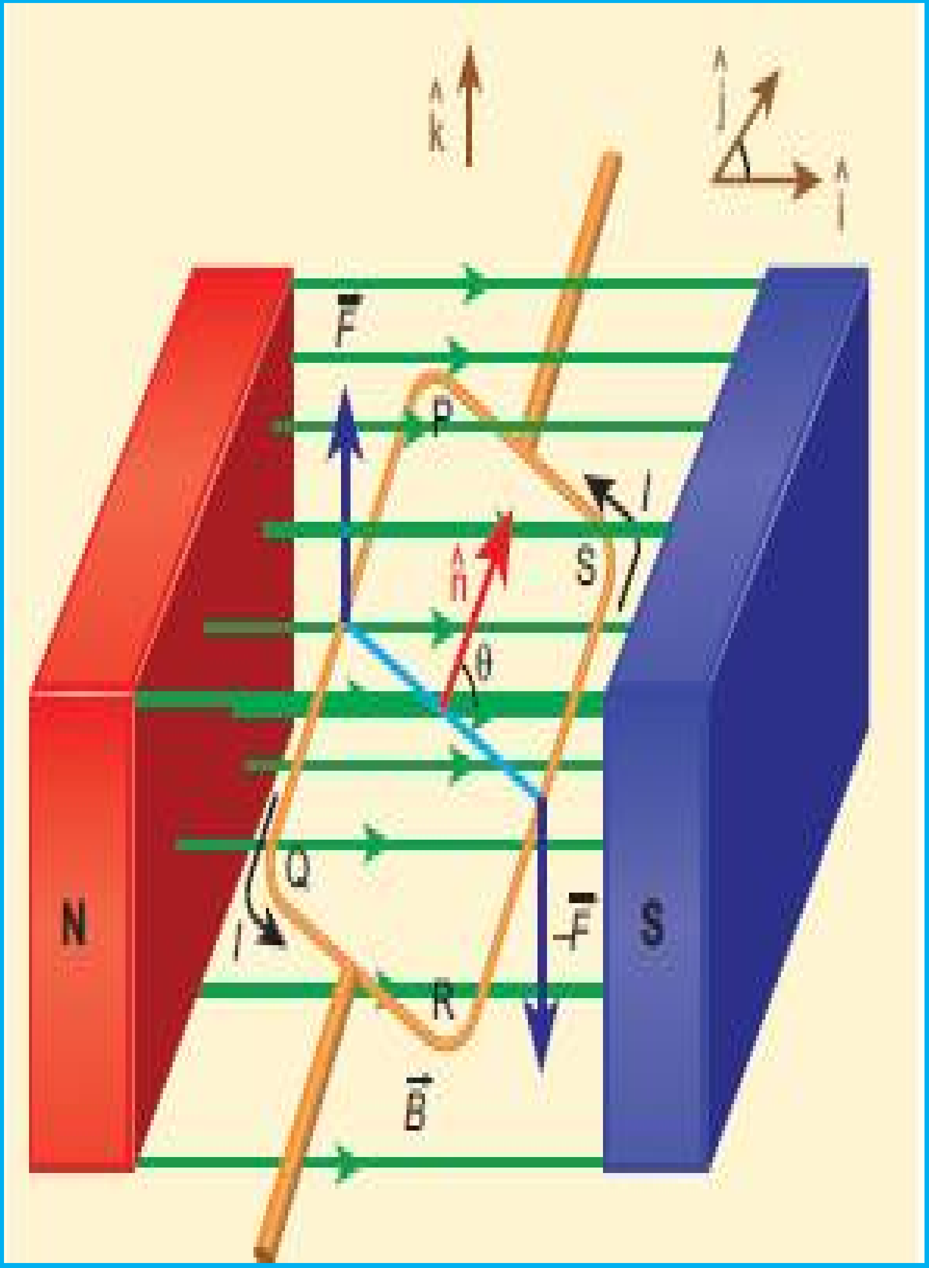
Since, the unit vector normal to the plane $\hat{n}$ is along the direction of $\hat{k}$.

(c) The force on section RS

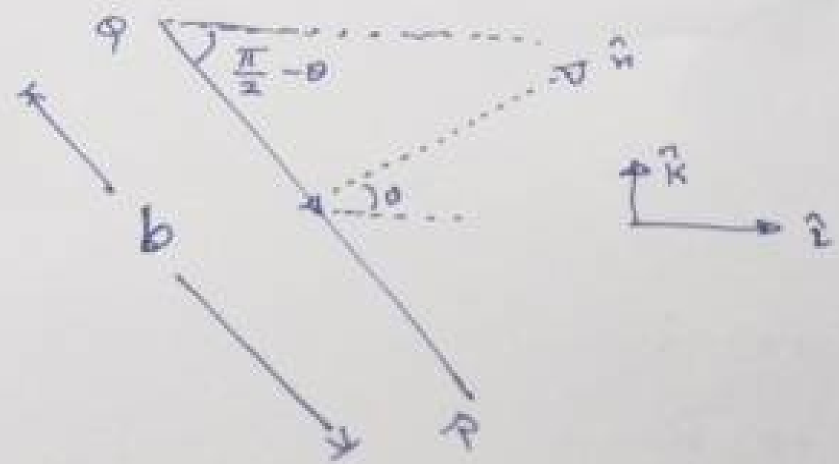
$$\vec{l} = a\,\check{j} and\, \vec{B} = B\hat{i}$$

$$\vec{F}_{RS} = I\vec{l} \times \vec{B} = IaB\left(\hat{j} \times \hat{i}\right) = -\mathrm{IaB}\,\hat{\mathrm{k}}$$

Since, the unit vector normal to the plane is along the direction of $-\hat{\mathrm{k}}$.



(b) The force on section QR



horizontal component of $\vec{QR}$ = $b\cos\left(\frac{\pi}{2}-\theta\right)\hat{\imath}$

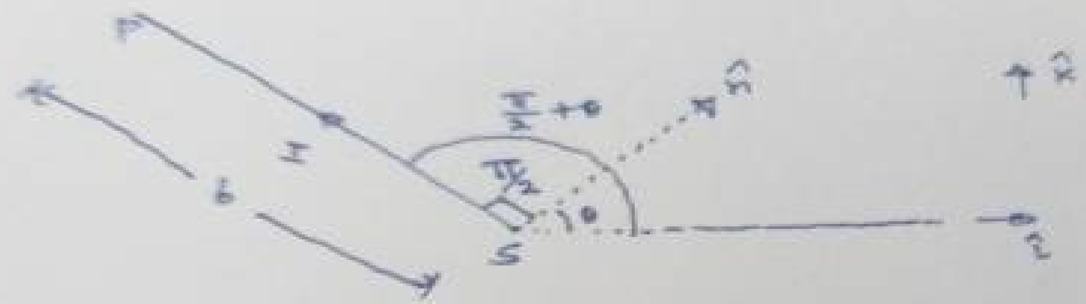
vertical component of $\vec{QR}$ = $b\sin\left(\frac{\pi}{2}-\theta\right)(-\hat{k})$

$$\vec{l} = b\cos\left(\frac{\pi}{2}-\theta\right)\hat{\imath} - b\ \sin\left(\frac{\pi}{2}-\theta\right)\hat{k}$$

$$\vec{B} = B\hat{\imath}$$

$$\vec{F}_{QR} = I\ \vec{l}\times\vec{B} = -\,IbB\sin\left(\frac{\pi}{2}-\theta\right)\hat{j}$$

$$\vec{F}_{QR} = -IbB\cos\theta\ \hat{j}$$



horizontal component of $\overrightarrow{PS} = b\cos\left(\frac{\pi}{2}+\theta\right)\hat{i}$

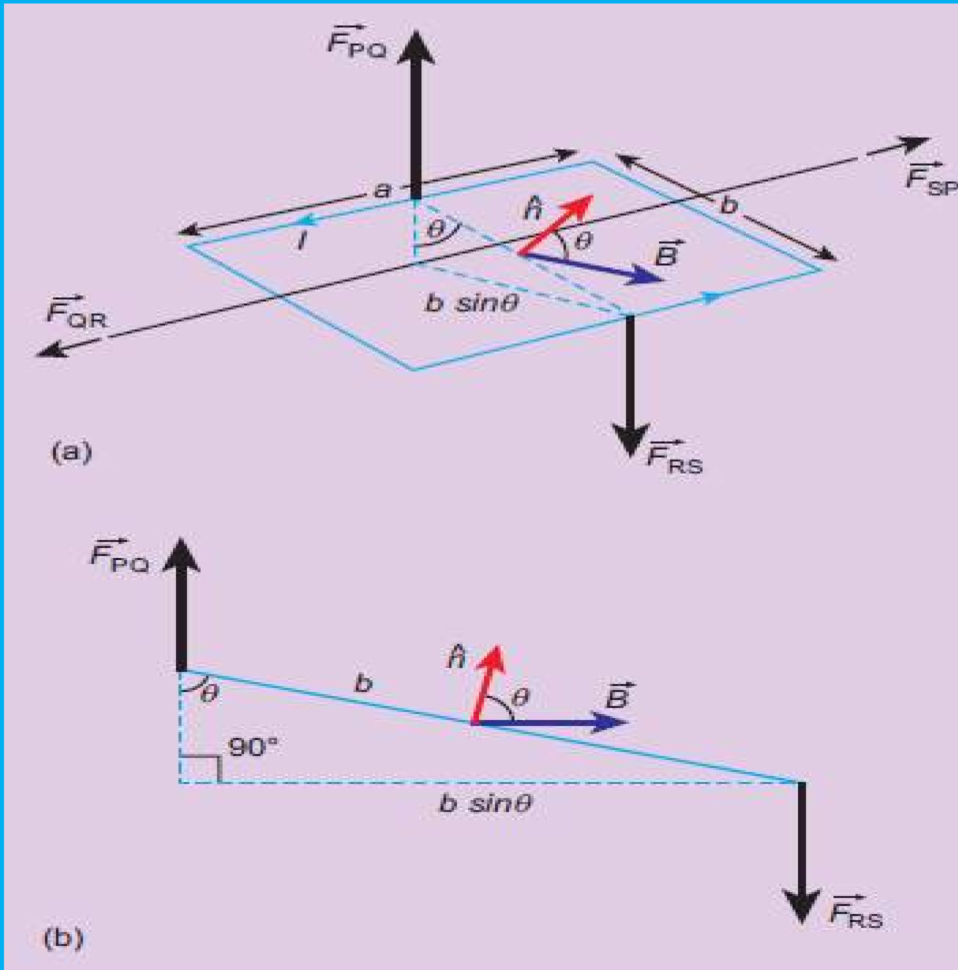
vertical component of $\overrightarrow{PS} = b\sin\left(\frac{\pi}{2}+\theta\right)\hat{k}$

(d) The force on section SP

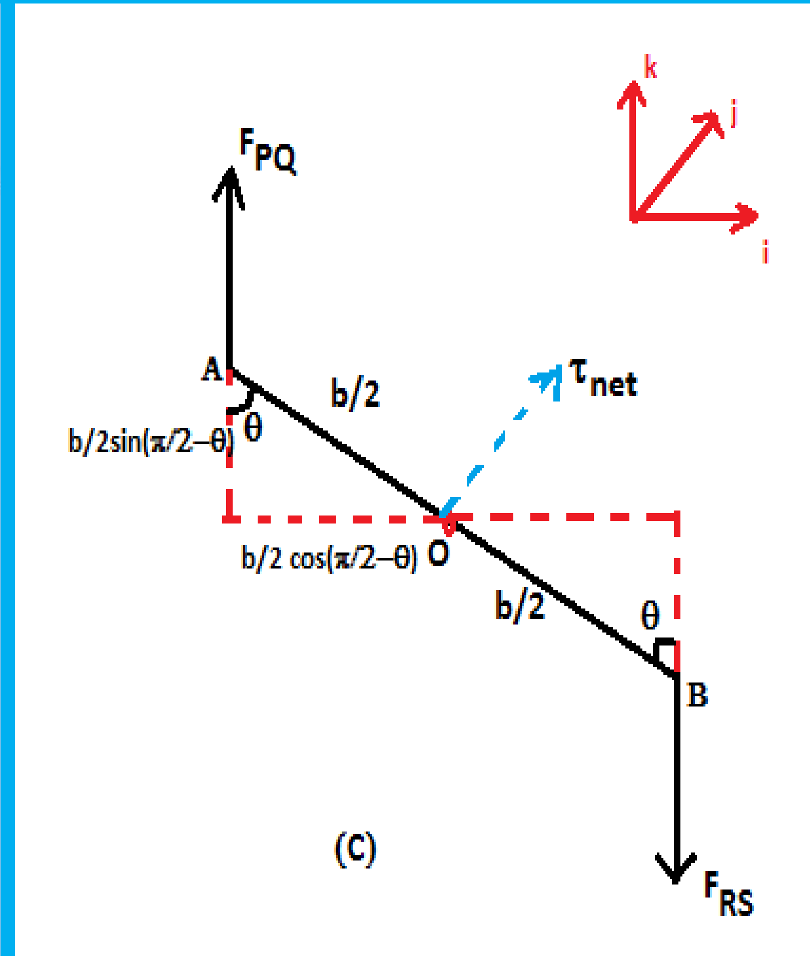
$$\vec{l} = b\cos\left(\frac{\pi}{2}+\theta\right)\hat{i} + \sin\left(\frac{\pi}{2}+\theta\right)\hat{k} \; and \; \vec{B} = B\,\hat{i}$$

$$\vec{F}_{SP} = I\vec{l}\times\vec{B} = IbB\sin\left(\frac{\pi}{2}+\theta\right)\hat{j}$$

$$\vec{F}_{SP} = IbB\cos\theta\,\hat{j}$$



$\vec{F}_{PQ}$
a
b
$\vec{F}_{SP}$
$\hat{n}$
θ
I
$\vec{B}$
$\vec{F}_{QR}$
$b \sin\theta$
$\vec{F}_{RS}$
(a)
$\vec{F}_{PQ}$
$\hat{n}$
b
θ
$\vec{B}$
90°
$b \sin\theta$
$\vec{F}_{RS}$
(b)



k
j
i
F_{PQ}
A
b/2
τ_{net}
θ
b/2sin(π/2−θ)
b/2 cos(π/2−θ)
O
b/2
θ
B
(C)
F_{RS}

$$\overrightarrow{OA} = \frac{b}{2}\cos\left(\frac{\pi}{2} - \theta\right)(-\hat{i}) + \frac{b}{2}\sin\left(\frac{\pi}{2} - \theta\right)(-\hat{k})$$

$$= \frac{b}{2}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{k})$$

$$\overrightarrow{OB} = \frac{b}{2}\cos\left(\frac{\pi}{2} - \theta\right)(\hat{i}) + \frac{b}{2}\sin\left(\frac{\pi}{2} - \theta\right)(-\hat{k})$$

$$= \frac{b}{2}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{k})$$

$$\overrightarrow{OA} \times \vec{F}_{PQ} = \left\{\frac{b}{2}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{k})\right\} \times \{IaB\,\hat{k}\}$$

$$= \frac{1}{2} IabB\sin\theta\,\hat{j}$$

$$\overrightarrow{OB} \times \vec{F}_{RS} = \left\{\frac{b}{2}(\sin\theta\,\hat{i} + \cos\theta\,\hat{k})\right\} \times \{-IaB\,\hat{k}\}$$

$$= \frac{1}{2} IabB\sin\theta\,\hat{j}$$

The net torque $\vec{\tau}_{net} = IabB\sin\theta\,\hat{j}$

$$\vec{\tau}_{net} = baBI\sin\theta\,\hat{k} = ABI\sin\theta\,\hat{k}$$

The net torque $\vec{\tau}_{net} = IabB\sin\theta\,\hat{j}$

Cases:

(a) When $\theta = 90^\circ$, then the torque on the current loop is maximum which is

$$\vec{\tau}_{net} = abIB\,\hat{j}$$

Note here $\vec{p}_m$ points perpendicular to the magnetic field $\vec{B}$. The torque is maximum in this orientation.

(b) When $\theta = 0^\circ$ or 180° then the torque on the current loop is

$$\vec{\tau}_{net} = 0$$

when $\theta = 0^\circ$, $\vec{p}_m$ is parallel to $\vec{B}$ and for $\theta = 180^\circ$, $\vec{p}_m$, is anti - parallel to $\vec{B}$. The torque is zero in these orientations.